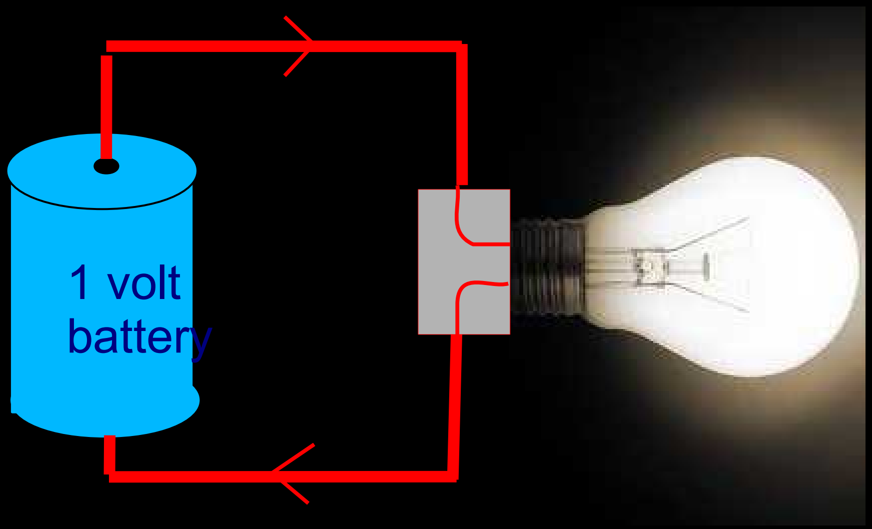
Nuclear Binding & Stability

Stanley Yen
TRIUMF

UNITS: ENERGY

Energy measured in electron-Volts (eV)



1 volt battery boosts energy of electrons by 1 eV

1 e-Volt = 1.6×10^{-19} Joule

1 MeV = 10^{6} eV 1 GeV = 10^{9} eV

Recall that atomic and molecular energies ~ eV
nuclear energies ~ MeV

UNITS: MASS

From $E = mc^2$ $m = E / c^2$ so we measure masses in MeV/c^2

$1\ MeV/c^2 = 1.7827 \times 10^{-30}$ kg

Frequently, we get lazy and just set c=1, so that we measure masses in MeV

e.g. mass of electron = 0.511 MeV
mass of proton = 938.272 MeV
mass of neutron=939.565 MeV

Also widely used unit of mass is the atomic mass unit (amu or u)
defined so that Mass(^{12}C atom) = 12 u

$1\ u = 931.494\ MeV = 1.6605 \times 10^{-27}$ kg

nucleon = proton or neutron

NUCLEAR PHYSICS describes nucleon bound into nuclei.

Nuclei are labelled $^{A}_{Z}El_{N}$ **where**

El = the chemical symbol of the element

A = Mass Number, number of neutrons N + number of protons Z

$A = N + Z$

N= Number of neutrons

Z= Number of protons

For example: $^{7}_{3}Li$ **lithium-7, mass =7, protons=3 (hence N=4)**

ISOTOPE: nuclei with the same Z and different N (same element)

For ex.: $^{12}_{6}C$, $^{13}_{6}C$, ^{14}C, ^{18}C,.....

ISOBAR: nuclei with same A (different elements)

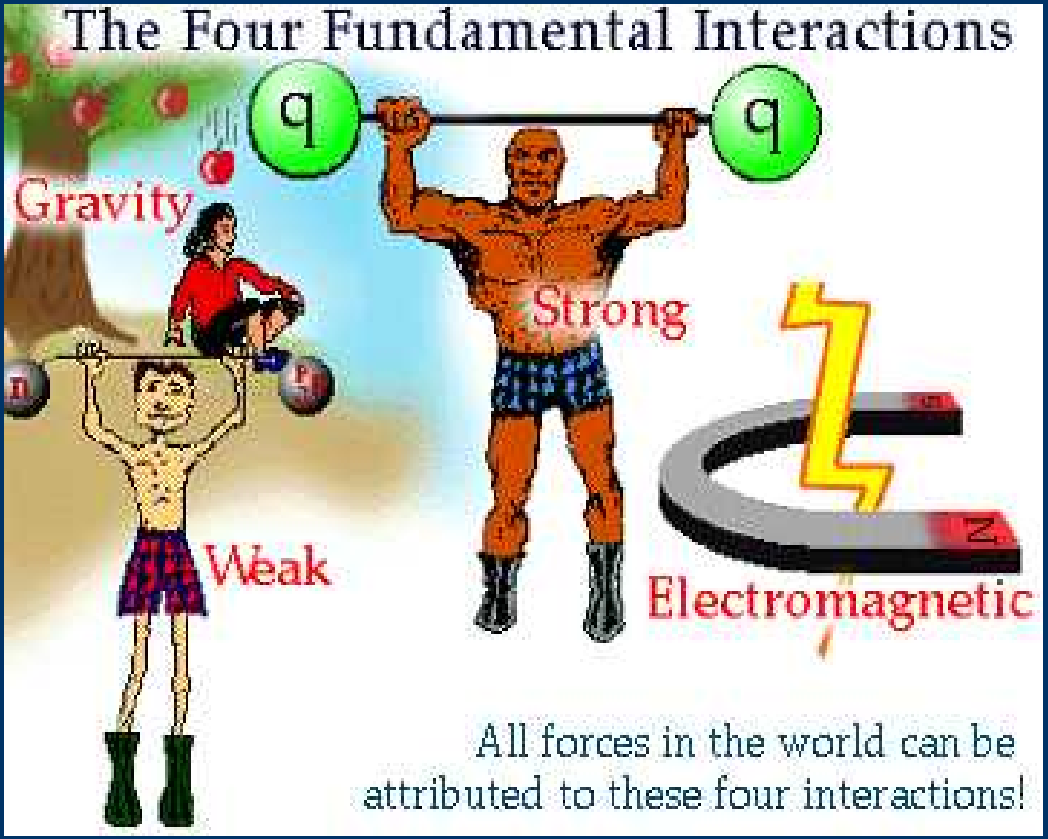
For ex.: ^{39}Ca, ^{39}K, ^{39}Ar

ISOTONE: nuclei with the same N (different elements)

For ex.: ^{39}Ca, ^{38}K, ^{37}Ar

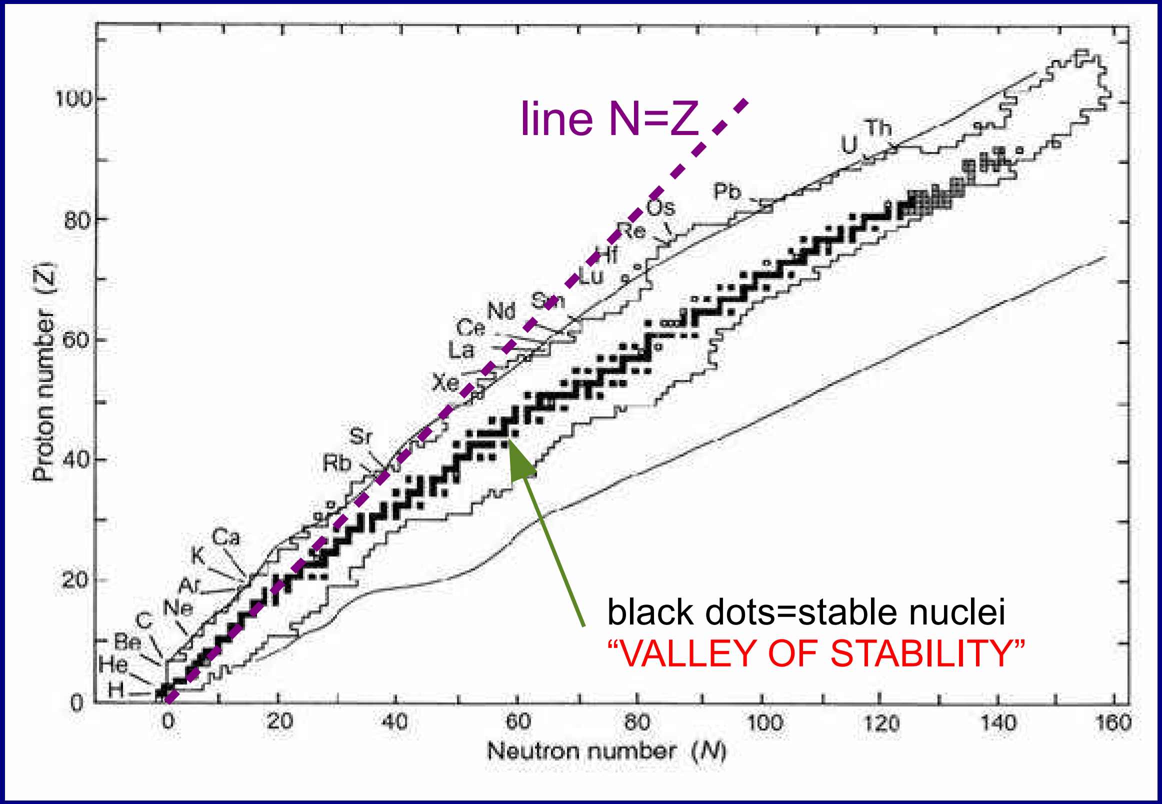
"nuclide" means one particular nuclear species, e.g. ^{7}Li and ^{56}Fe are two different nuclides

There are 4 fundamental types of forces in the universe.
1. Gravity – very weak, negligible for nuclei except for neutron stars
2. Electromagnetic forces – Coulomb repulsion tends to force protons apart)
3. Strong nuclear force – binds nuclei together; short-ranged
4. Weak nuclear force – causes nuclear beta decay, almost negligible compared to the strong and EM forces.



How tightly a nucleus is bound together is mostly an interplay between the attractive strong force and the repulsive electromagnetic force.

Let's make a scatterplot of all the stable nuclei, with proton number Z versus neutron number N.



Note: 1. Not all combinations of N, Z are possible!

2. For light nuclei, stable nuclei cluster around N=Z.

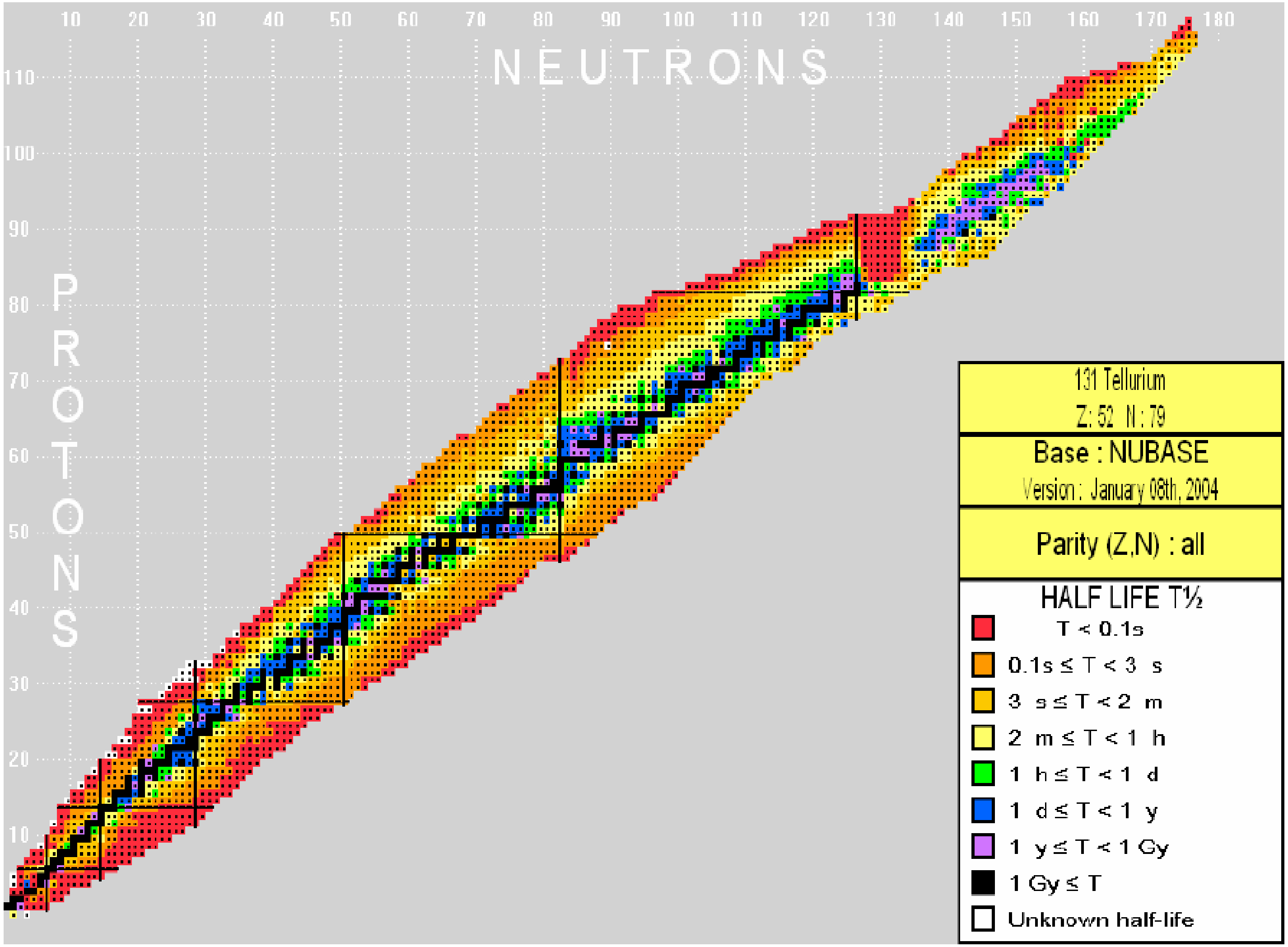
3. For heavier nuclei, N>Z ; the heavier the nucleus, the more neutrons you need to make the nucleus stable

e.g. ^{56}Fe has 26 p, 28 n but ^{208}Pb has 82 p, 126 n (nearly 50% more n than p)

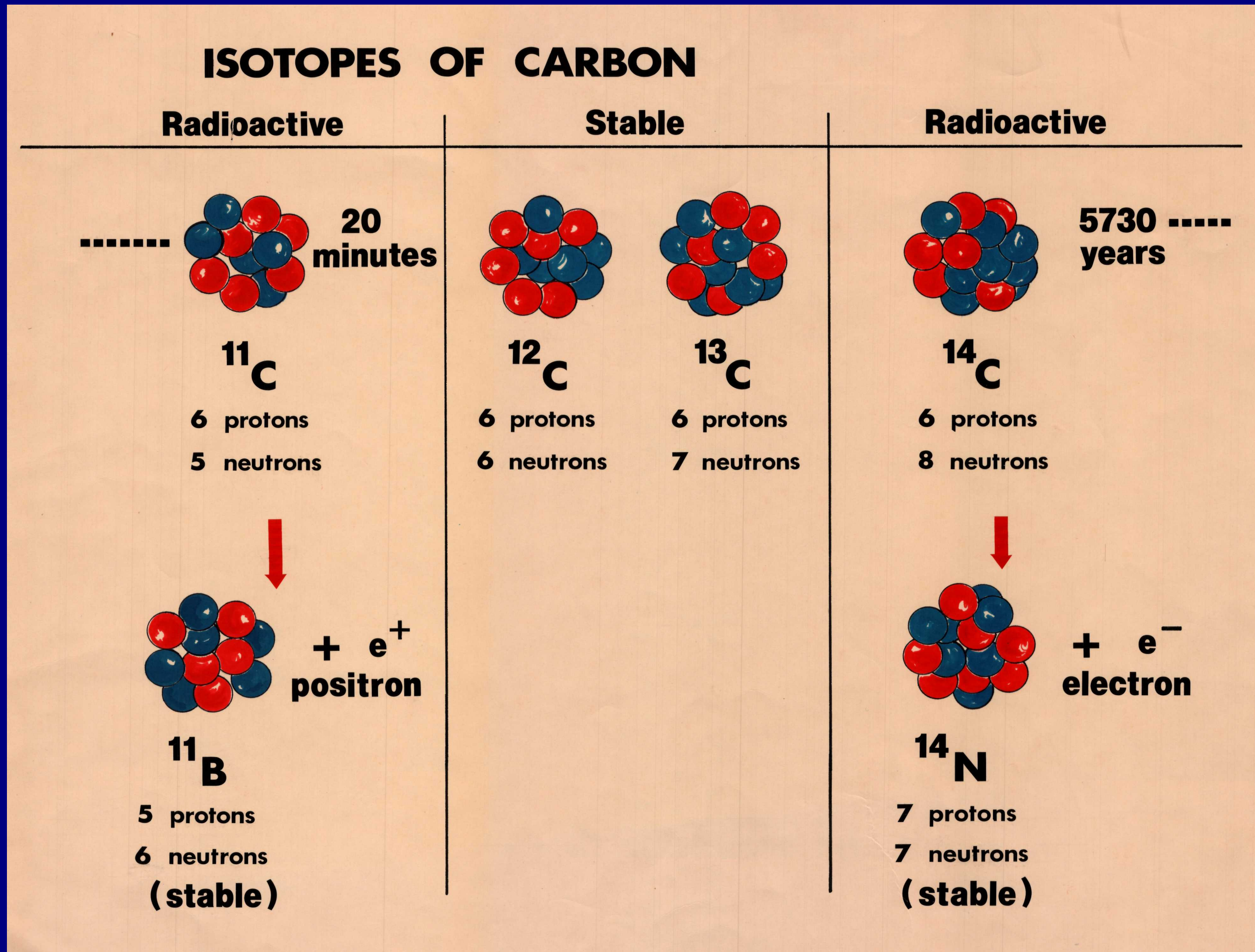
Reason: protons and neutrons both feel the attractive strong interaction, but only protons feel the repulsive Coulomb force.

CHART OF THE NUCLIDES

- As function of half-life



The further the nuclide is from the valley of stability, the shorter is its half-life.

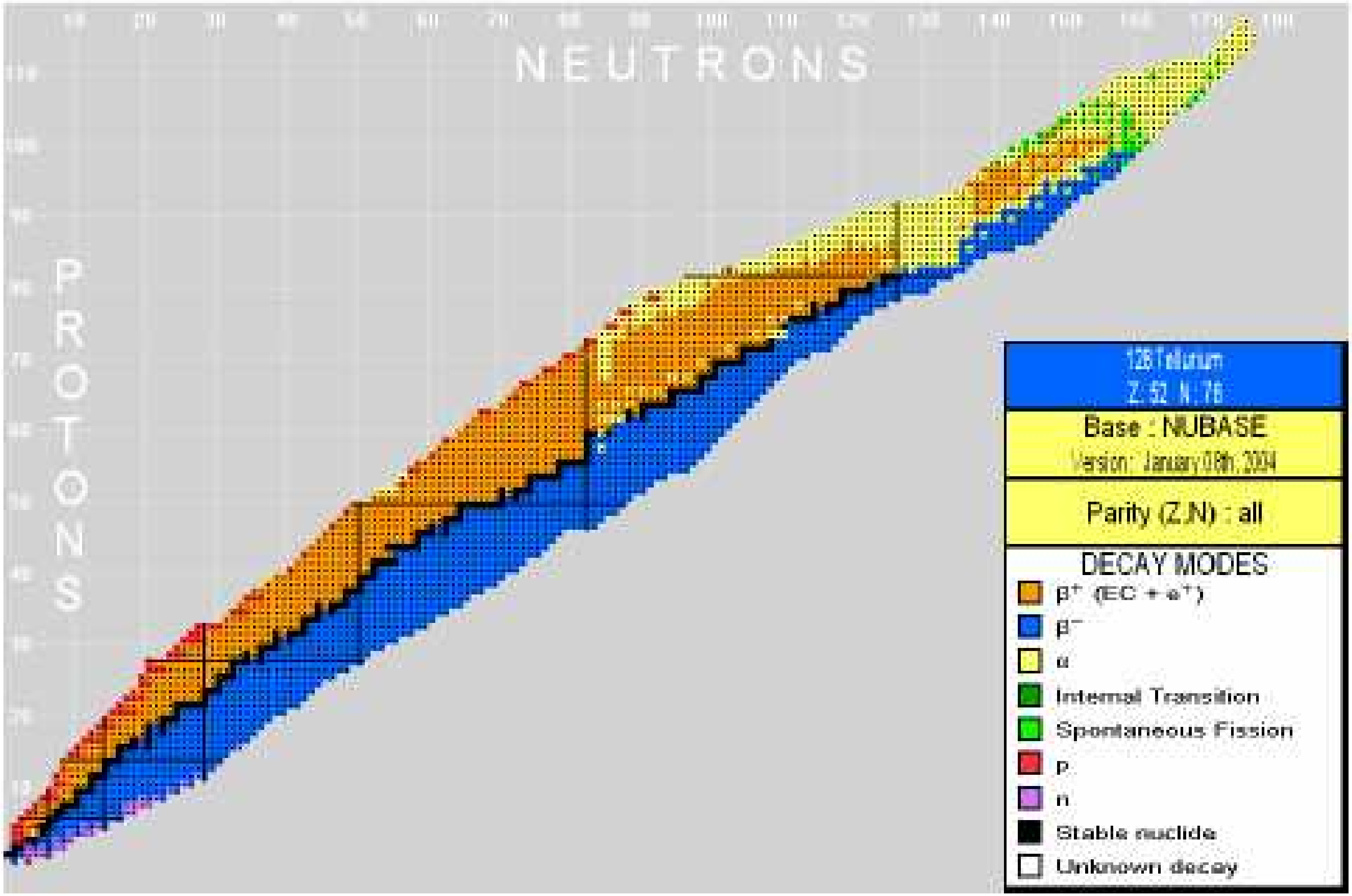


proton-excess side of valley of stability: nucleus sheds its excess protons by beta+ decay where $p \rightarrow n + e^{+} + \nu_e$

neutron-excess side of valley of stability: nucleus sheds its excess neutrons by beta- decay

where $n \rightarrow p + e^{-} + \overline{\nu}_e$

DECAY MODES OF NUCLEI



http://csnwww.in2p3.fr/amdc/

NUCLEAR MASSES & BINDING ENERGY

Recall Einstein's famous formula $E = mc^2$

High energy content E means higher mass m.
This is negligible on scales of everyday life.

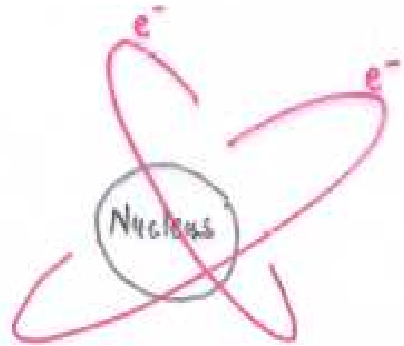
e.g suppose I expend energy ΔE = 1 Joule to wind up a 1 kg alarm clock
The clock's mass increases by $\Delta m = \Delta E / c^2 = 1.1\text{x}10^{-17}$ kg

i.e. the mass changes by ~ 1 part in 10^{17} (negligible)

As we will see shortly, the change in mass is
not negligible on the nuclear scale.

When a system of two or more particles get bound to each other,
the energy (and hence the mass) of that system decreases.

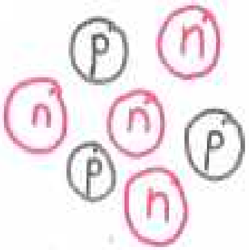
Atom:



$$m(\text{atom}) = m(\text{nucleus}) + Z\,m_e - b/c^2$$

where b = binding energy of the electrons

Nucleus:

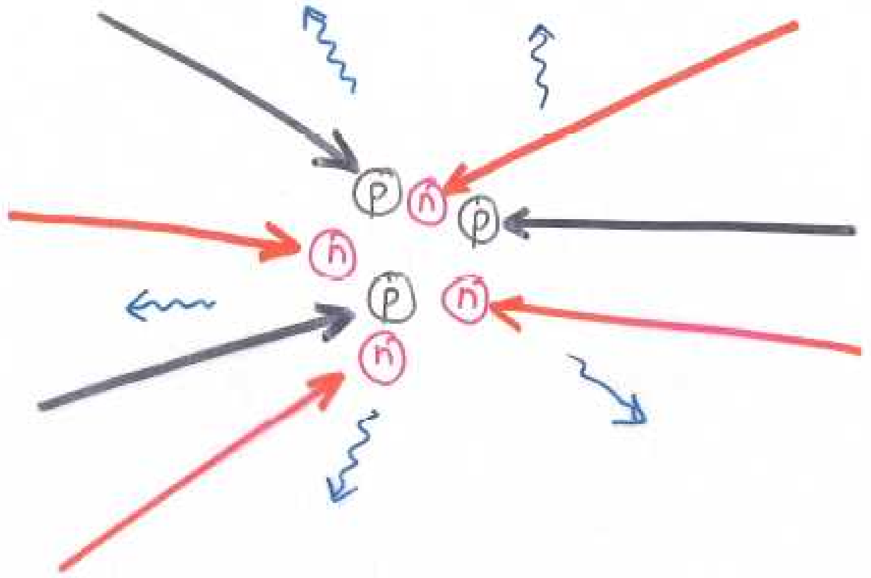


$$m(\text{nucleus}) = Z\,m_p + N\,m_n - B/c^2$$

where B = nuclear binding energy

What does nuclear binding energy mean?

Suppose we assemble a nucleus from Z protons and N neutrons, initially at infinite separation



nuclear binding energy B is the amount of energy given off when the nucleus is assembled

Similarly B is the energy required to tear the nucleus apart into Z protons and N neutrons at infinity

> The larger the binding energy, the smaller the mass of the nucleus.

Relative sizes of atomic + nuclear binding energies

Consider simplest atom (hydrogen)

atomic binding energy = 13.6 eV

Consider simplest nucleus (deuteron)

nuclear binding energy = 2.22 MeV

∴ nuclear binding energies >> atomic binding energies

$$m\,(\text{atom}) = Z\,(m_p + m_e) + N\,m_n - B/c^2 - b/c^2$$

Neglecting the small atomic binding energy b

$$m\,(\text{atom}) = Z\,m\,(^1H) + N\,m_n - B/c^2$$

↑ atomic hydrogen

Data tables typically have atomic masses, not nuclear masses

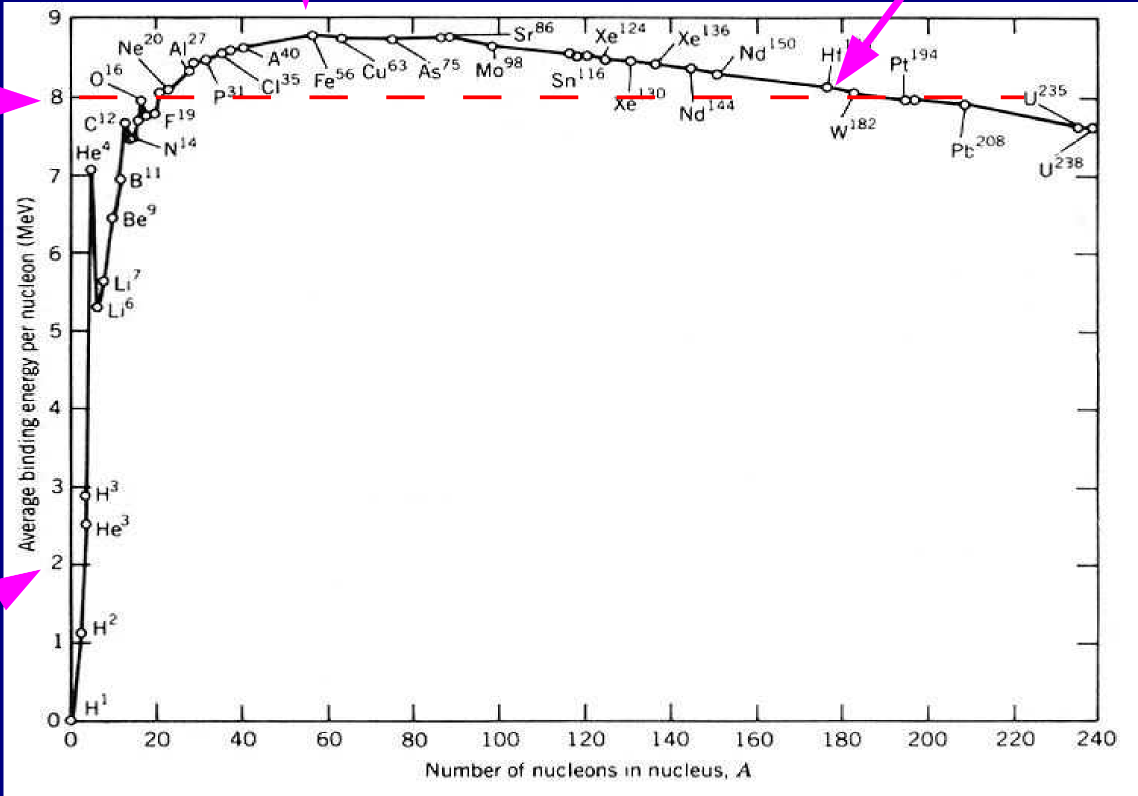
e.g. "mass of ^{19}F" means ^{19}F atom.

Now let's consider B /A i.e. the average binding energy per nucleon in a nucleus.

tightest binding for **Fe region nuclei**

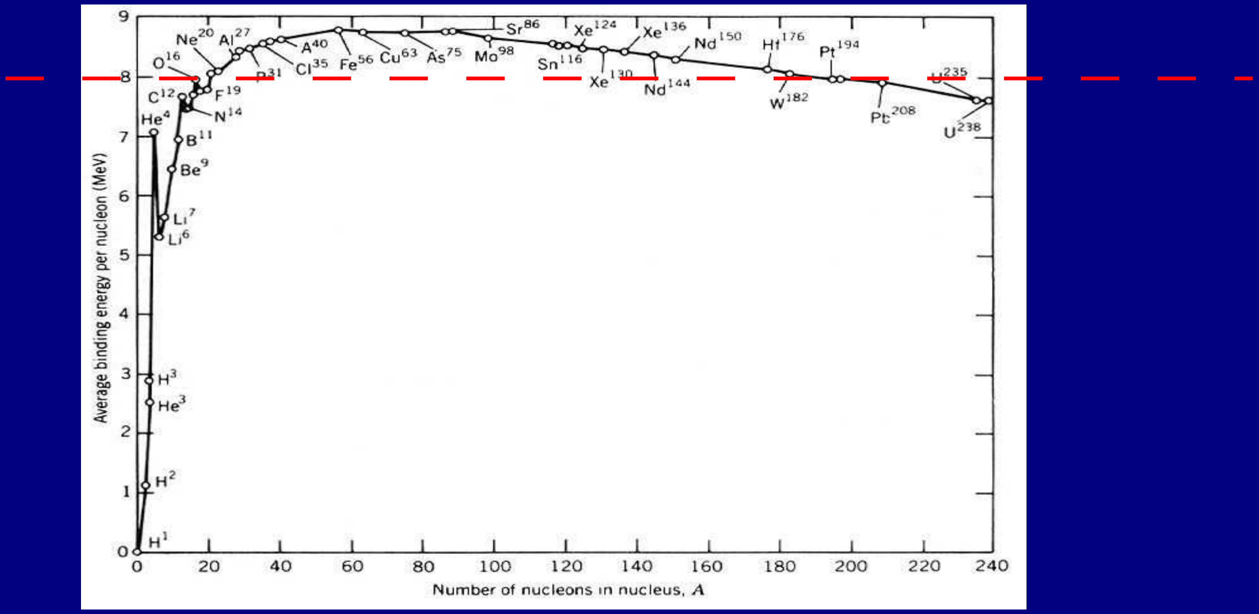
gradual decrease in binding with increasing mass, due to greater Coulomb repulsion between the protons

average binding energy ~ 8 MeV per nucleon – almost constant for mass 12 to mass 238



initial rapid rise as more nucleons added

This behaviour, that the B/A value is almost a constant for all but the lightest nuclei, is termed <u>the saturation of nuclear forces</u>. It is a consequence of the short range of the strong nuclear force: each nucleon feels the attraction of only its nearest neighbours.

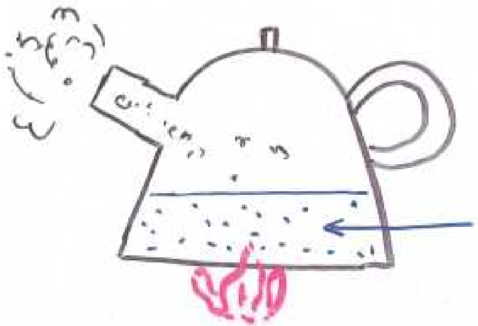


If each of the A nucleons in a nucleus could bind to each of the other (A-1) nucleons, then the total binding energy B would be proportional to the number of pairs, ie. $B \sim A(A-1) \sim A^2$,
and $B/A \sim A$. This is NOT what is observed in nuclei !

Instead, $B/A \sim$ constant. This indicates that nuclear binding forces must be short-ranged, as shown by analogy in the next slide.

Mundane example of saturation of short range forces

The inter-molecular forces binding water molecules to each other in liquid form are weak, short range Van der Waals forces. Each water molecule feels only its nearest neighbours.



Let the number of water molecules = A

When the water is heated up, the inter-molecular bonds are broken and the water evaporates as steam.

$$\begin{pmatrix}\text{Energy required}\\ \text{to evaporate}\\ \text{A molecules}\\ \text{of water}\end{pmatrix} = \begin{matrix}\text{Binding}\\ \text{Energy}\\ B\end{matrix} \propto A$$

i.e. $\frac{B}{A}$ = constant = 10·5 kilo calories/mole

The Van der Waals inter-molecular forces have short range compared to the size of the pot of water.

On the other hand, consider a large sphere of water of mass M bound together by the force of gravity

$$\begin{pmatrix} \text{gravitational} \\ \text{binding} \\ \text{energy } B \end{pmatrix} = \frac{3}{5}\frac{GM^2}{R}$$

Since $M \propto A$ (the number of water molecules)

$$B \propto A^2$$

$$B/A \propto A$$

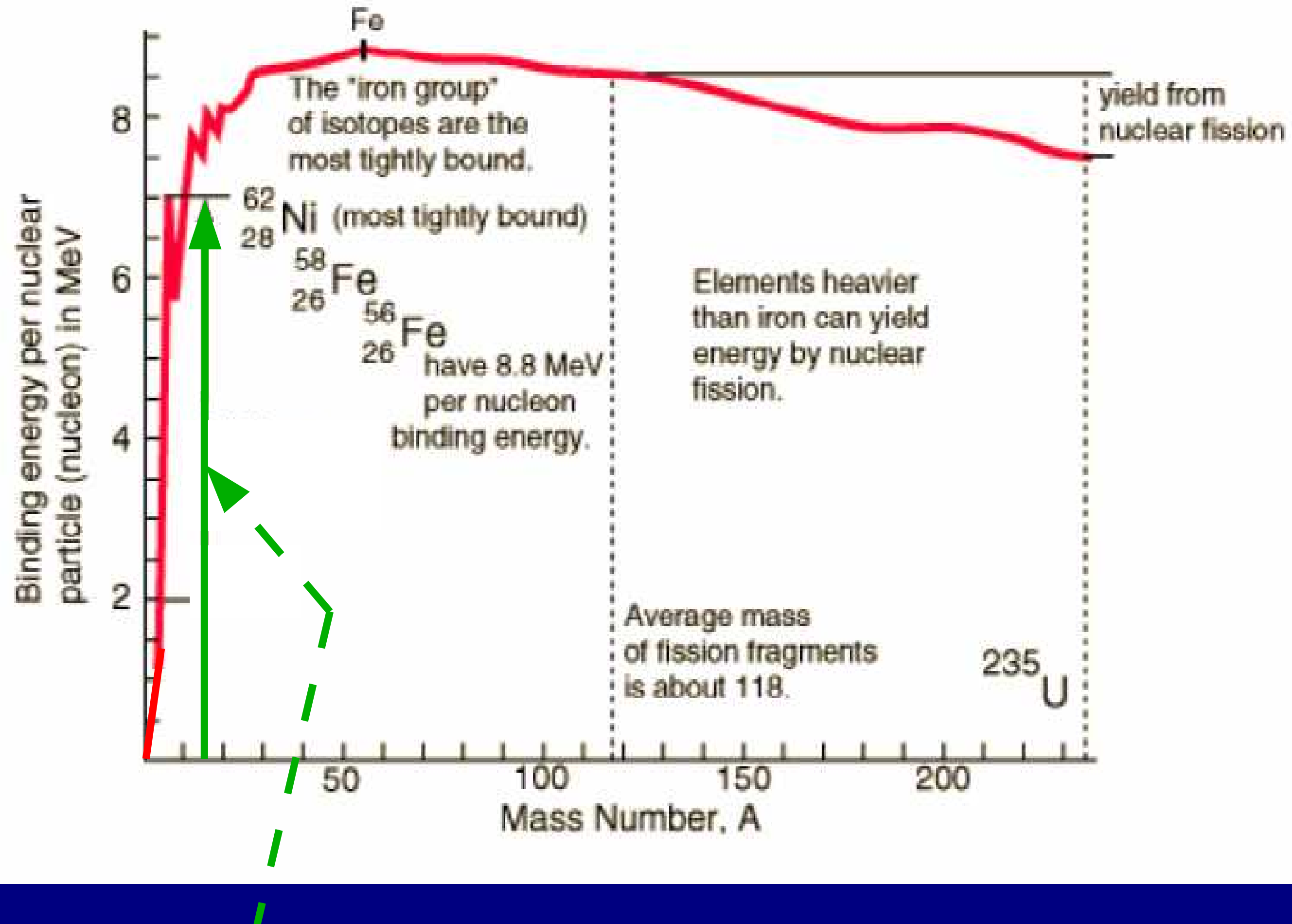
The gravitational binding energy does not saturate because each water molecule feels the gravitational attraction of every other molecule, not just its nearest neighbours.

This is because gravity is an infinite range force.

The average binding energy of ~ 8 MeV per nucleon is nearly 1% of the mass of a proton or neutron (938 MeV).

i.e. the mass of a nucleus is nearly 1% smaller than the mass of its constituent nucleons, because of the large binding energy. Easily measured, and not negligible as it is in atoms and molecules.

Fission and fusion can yield energy

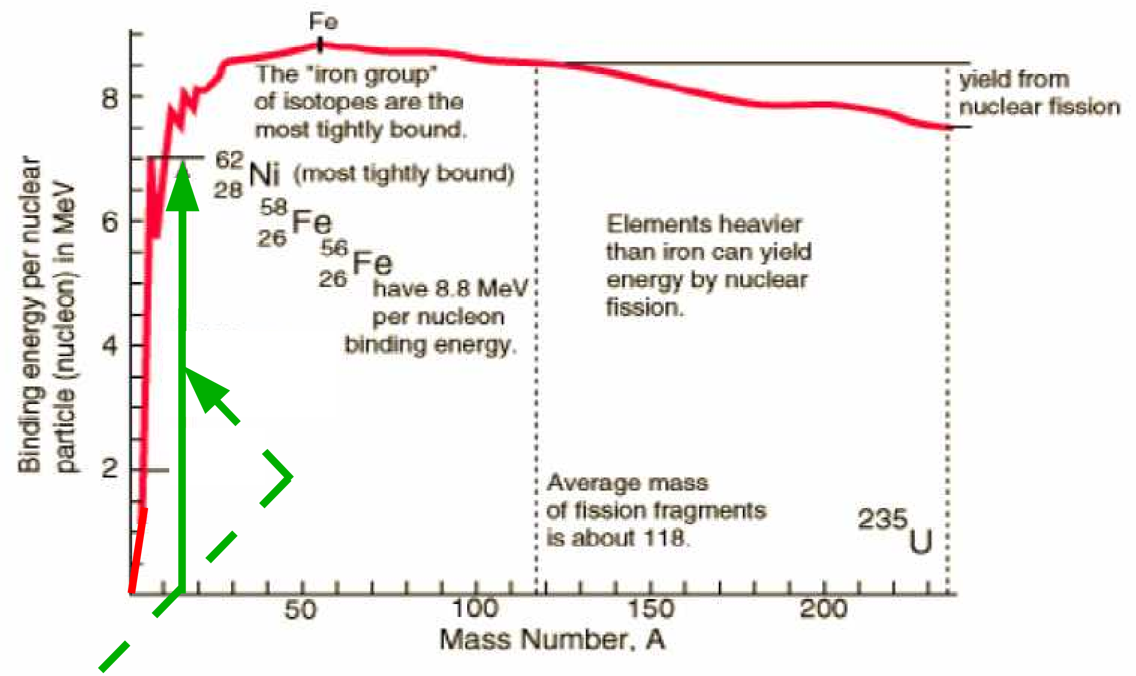


going from A=1 to A=4, the average binding energy per nucleon increases from 0 to 7 MeV

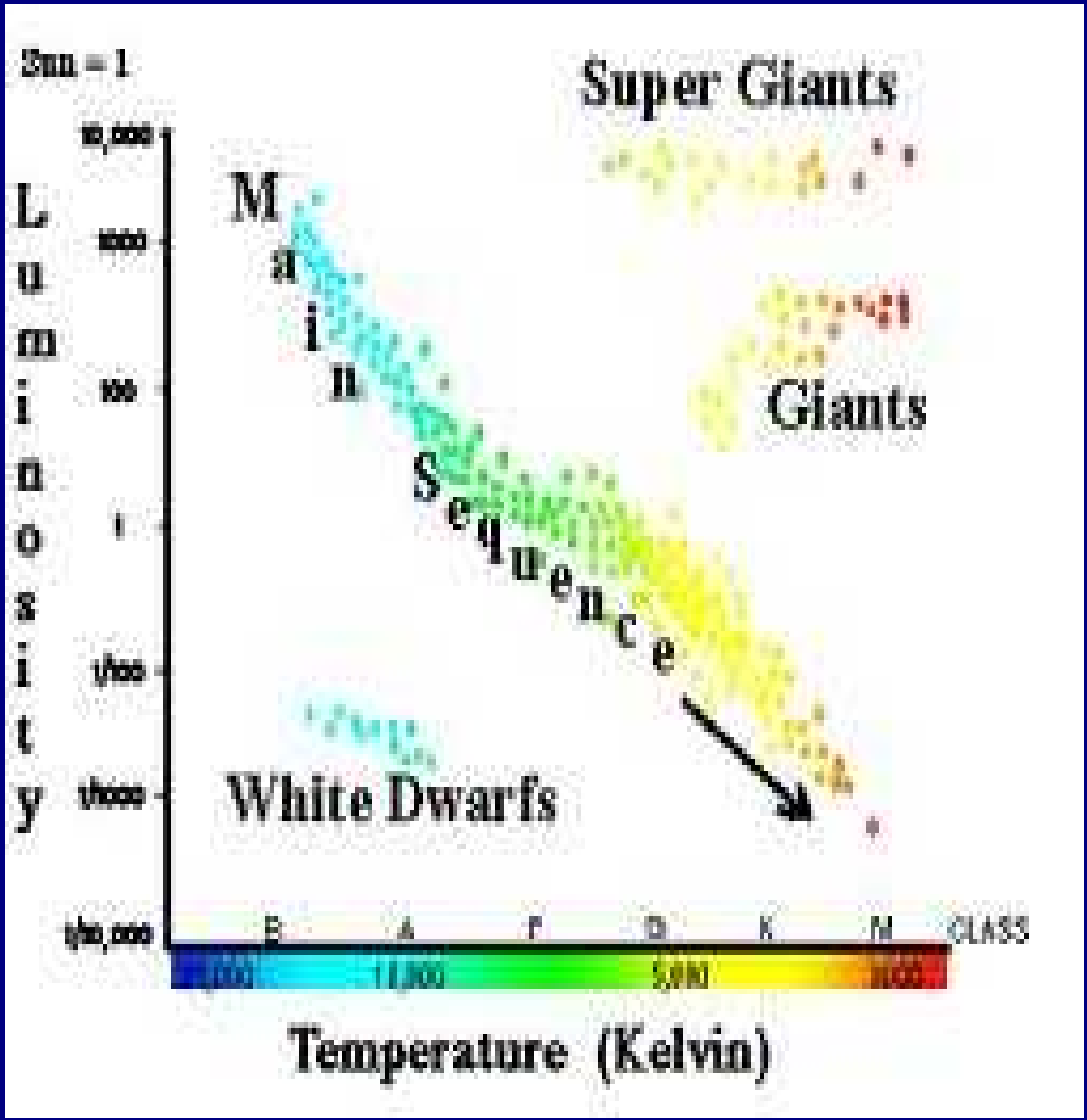
Fusion reaction 4 p $\rightarrow$ $^4He + 2\ e^+ + 2\ \nu_e$ liberates ~ 4 x 7 = 28 MeV

THIS REACTION PRODUCES ENERGY IN THE SUN!

Fission and fusion can yield energy



hydrogen→helium gives the biggest gain in binding energy. Stars spend most of their lives in this stage. (Main sequence stars). Later stages which fuse He+He+He→C, He+C→O, etc. produce far less energy and last much shorter periods of time.

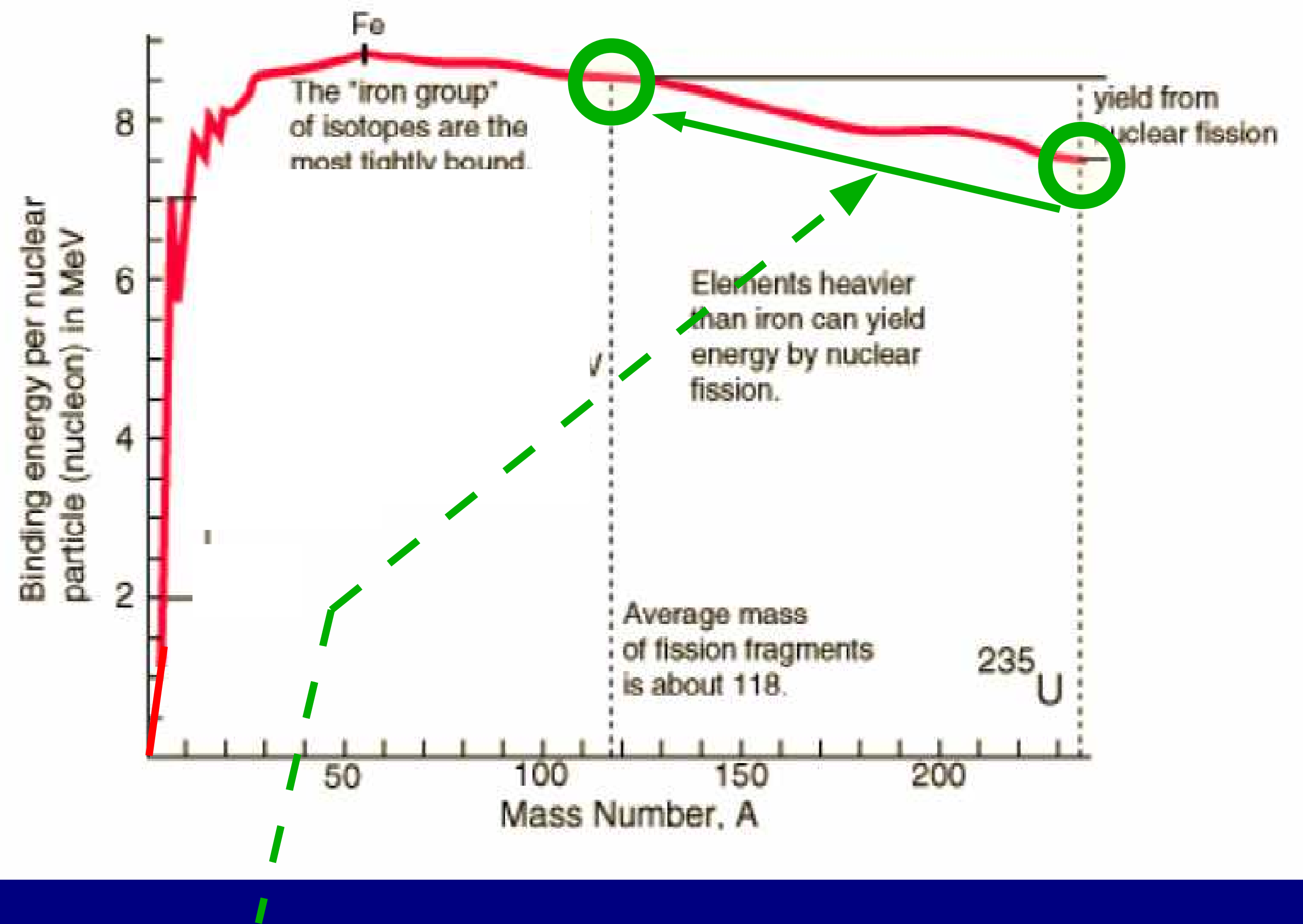


Sun = 1
Luminosity
Super Giants
Giants
Main Sequence
White Dwarfs
B A F G K M CLASS
5,000
Temperature (Kelvin)

Timeline for a 25 solar mass star

Hydrogen burning	7 Myr
Helium burning	500 kyr
Carbon burning	600 yr
Neon burning	1 yr
Silicon burning	1 day
Core collapse	<1 second

Fission and fusion can yield energy



going from A=235 to A=118, the average binding energy per nucleon increases by ~ 1 MeV

Fission of ^{235}U into 2 equal fragments gives about 235 x 1 = 235 MeV.
THIS REACTION PRODUCES ENERGY IN NUCLEAR REACTORS.

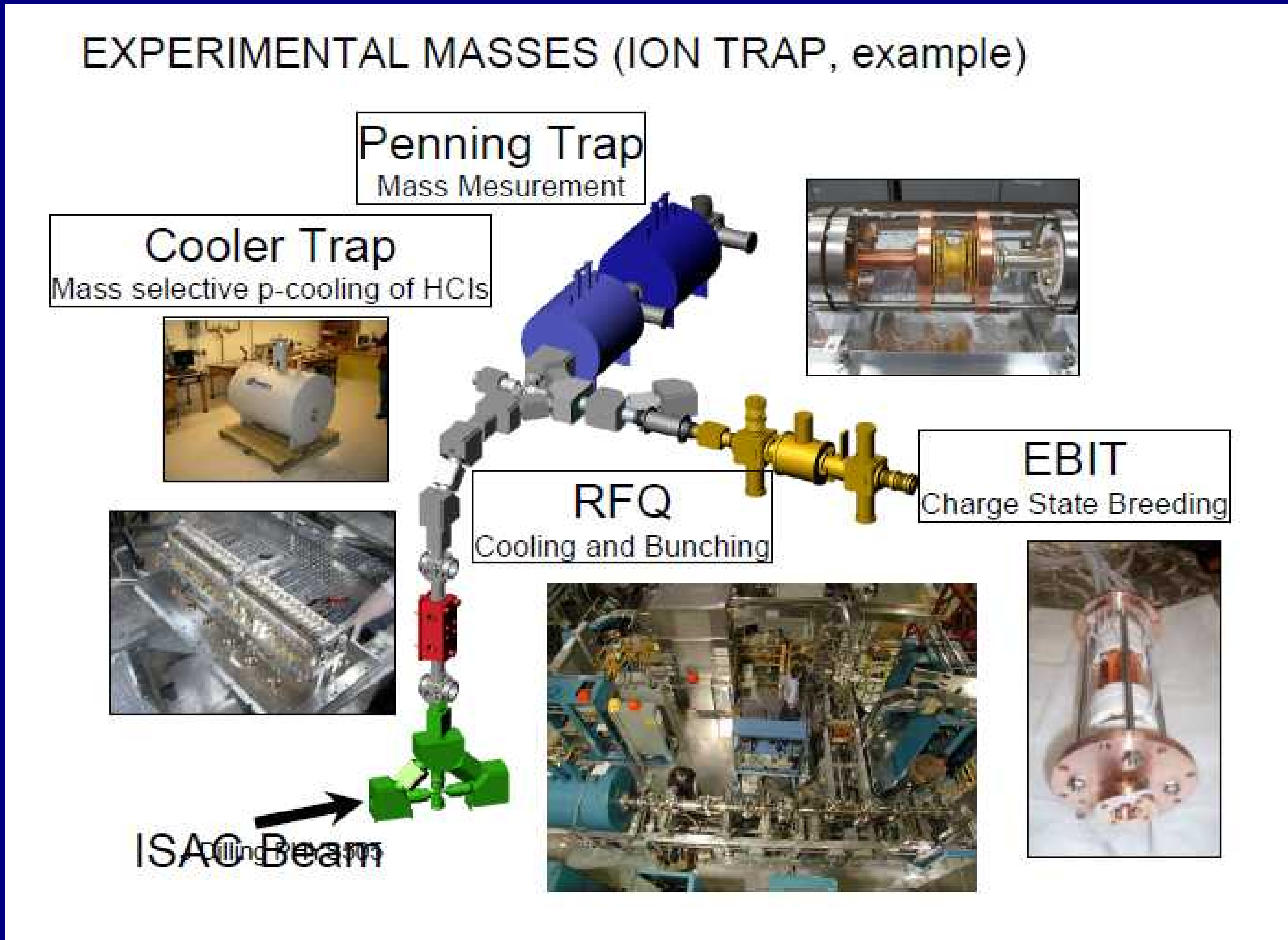
Binding energy B is experimentally determined by measuring the mass of the atom and then using the relationship

$$M(\text{atom}) = Z\, M(\text{hydrogen atom}) + N\, M(\text{neutron}) - B / c^2 \; - \; b / c^2$$

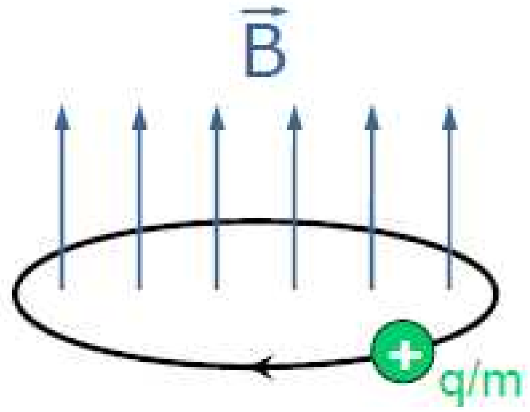
nuclear binding energy

binding energy of atomic electrons

TITAN facility at TRIUMF measures the masses of nuclides far from the valley of stability



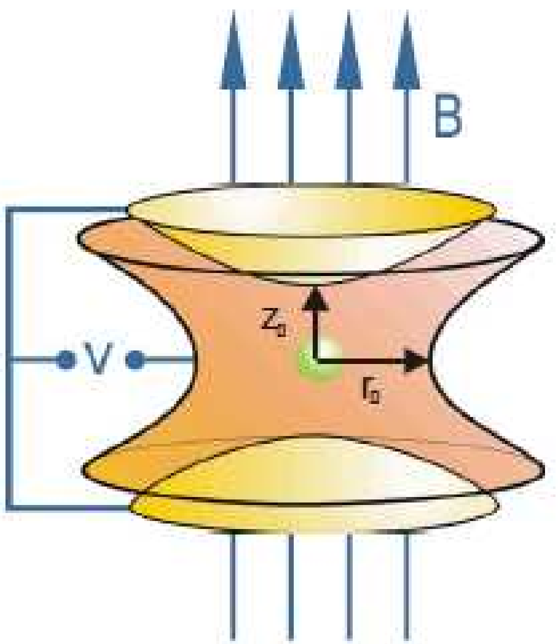
EXPERIMENTAL MASSES (ION TRAP, multi-turn)

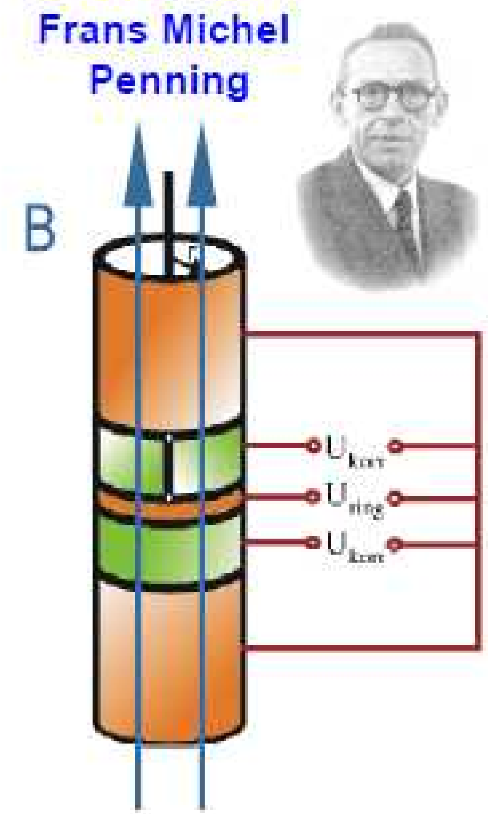


Cyclotron frequency: $\nu_c = \frac{1}{2\pi}\frac{q}{m}B$

Superposition
- strong homogeneous magnetic field
- weak electrostatic quadrupole field

PENNING trap





Liquid Drop Model of the Nucleus

The behaviour of B/A ~ constant is reminiscent of the behaviour of a liquid. In the 1930"s Von Weiszacker developed a model for the binding energy of a nucleus by modeling it as a drop of liquid with electric charge, plus some correction terms.

Like the water in our kettle, the binding energy has a term which is just proportional to the volume of liquid

i.e.

Volume Term $a_V A$

The Surface Term $-a_s A^{2/3}$

- Nucleons at the surface are surrounded by fewer other nucleons, thus the binding energy is reduced (-) compared to the nucleons further inside
- This contribution is proportional to the surface ~ $A^{2/3}$

The Coulomb Term $-a_c \frac{Z^2}{A^{1/3}}$

- The electrostatic repulsion between protons will further reduce (-) the binding energy
- This contribution is proportional to

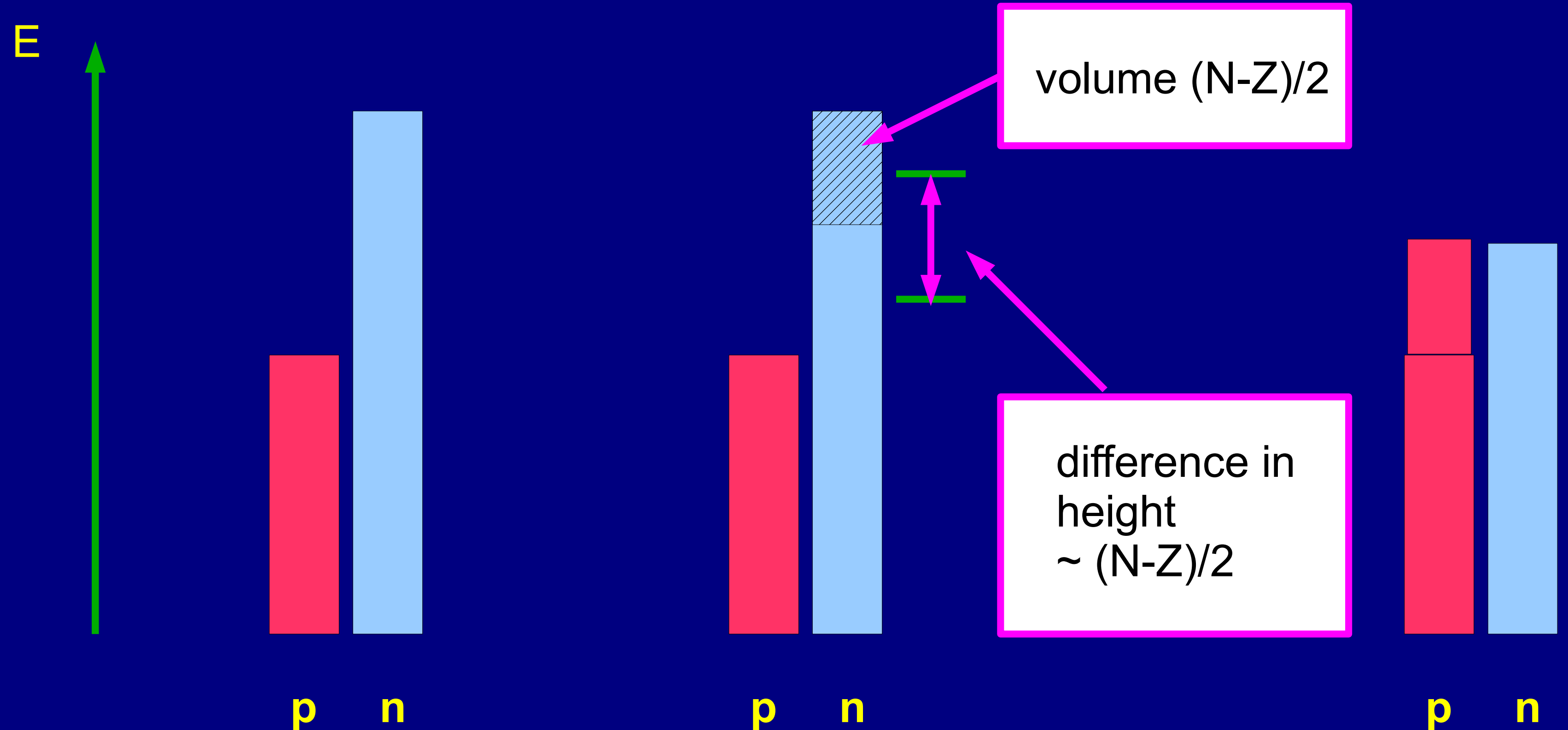
$$E_{\text{Coulomb}} = \frac{3}{5}\frac{Z(Z-1)\alpha\hbar c}{R}$$

- This is roughly proportional to $\frac{Z^2}{A^{1/3}}$

The (A)Symmetry Term $-a_a \frac{(N-Z)^2}{4A}$

- For small mass number stable nuclei tend to have the same number of protons and neutrons. Heavier nuclei accumulate more and more neutrons, to partially compensate the increasing Coulomb repulsion
- The symmetry energy arises because Pauli principle makes it more expensive (-) to have more nucleons of one type of nucleons than the other

Recall that protons and neutrons are fermions so like electrons in an atom, you can't have more than one of them in the same quantum state. Analogy with two columns of fluid with different heights – it is energetically favourable to let half the excess of blue fluid flow into the red fluid to minimize the gravitational potential energy.



The PairingTerm $-\frac{a_\delta}{A^{1/2}}$

- Nuclei are more stable when they have an even number of protons and/or neutrons. This observation is interpreted as a coupling of protons and neutrons in pairs
- The pairing energy depends on the mass number, empirically described as above

Empirical evidence of pairing in nuclei:
Of all the known stable nuclei 167 are even-N even-Z
4 are odd-N odd-Z

There is extra stability in the nucleus when there are an even number of protons, paired off, and an even number of neutron, paired off.

The binding energy can be parameterised using five terms:

$$B(A,Z) = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_a \frac{(N-Z)^2}{4A} - \frac{\delta}{A^{1/2}}$$

$$\begin{aligned} a_v &= 15.67\ MeV/c^2 \\ a_s &= 17.23\ MeV/c^2 \\ a_c &= 0.714\ MeV/c^2 \\ a_a &= 93.15\ MeV/c^2 \\ \delta &= -11.2\ MeV/c^2 && \text{even Z and N} \\ & \quad\ 0\ MeV/c^2 && \text{odd A} \\ & \quad +11.2\ MeV/c^2 && \text{odd Z and N} \end{aligned}$$

The semi-empirical mass formula, first introduced in 1935 by Weizsäcker (therefore also Weizsäcker-formula), can be written as

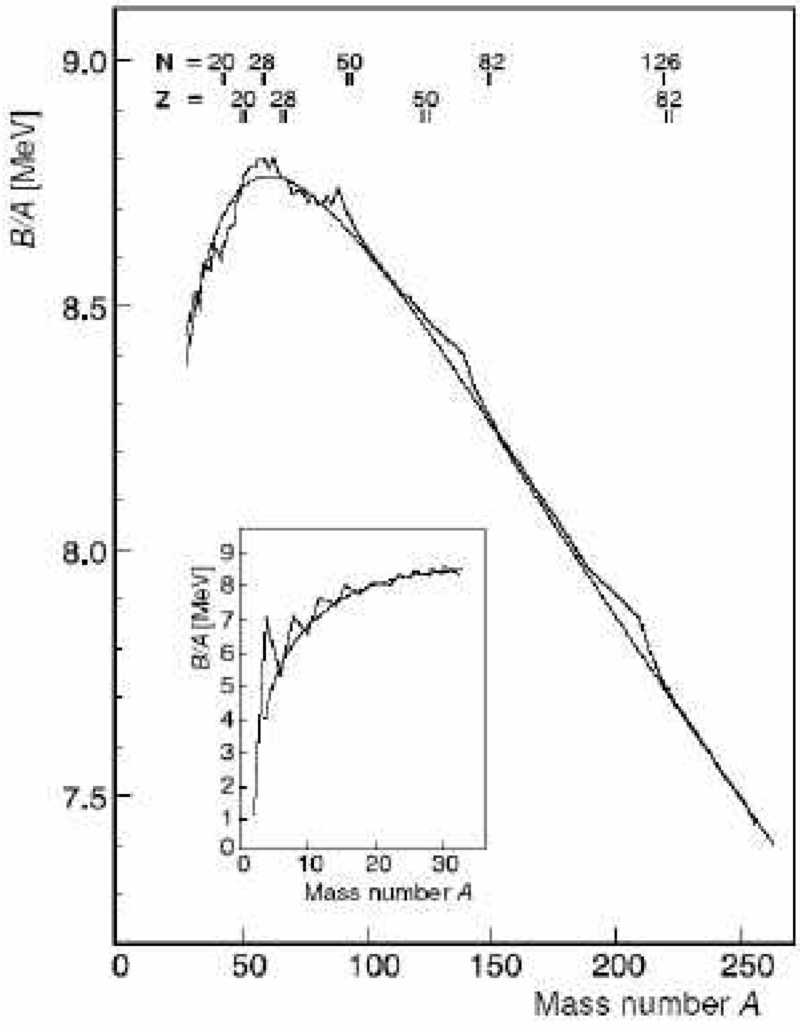
$$M(A,Z) = Nm_n + Zm_p + Zme - B(A,Z)$$

where $B(A,Z)$ is the binding energy.

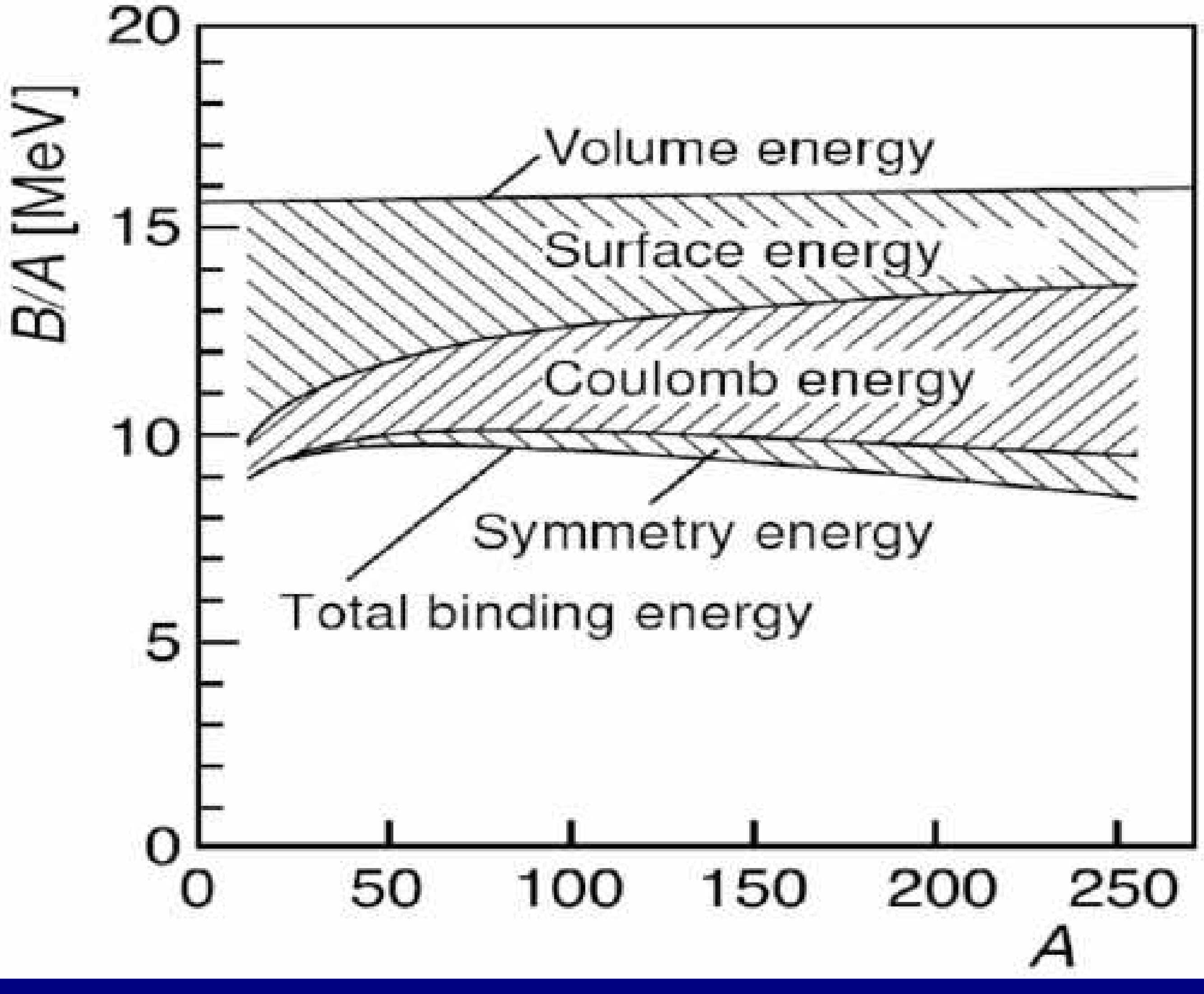
A very simple model of the nucleus which gives a pretty good explaination for the gross features of the nuclear binding energy.

How good? Let's compare the predictions of this model with experiment:

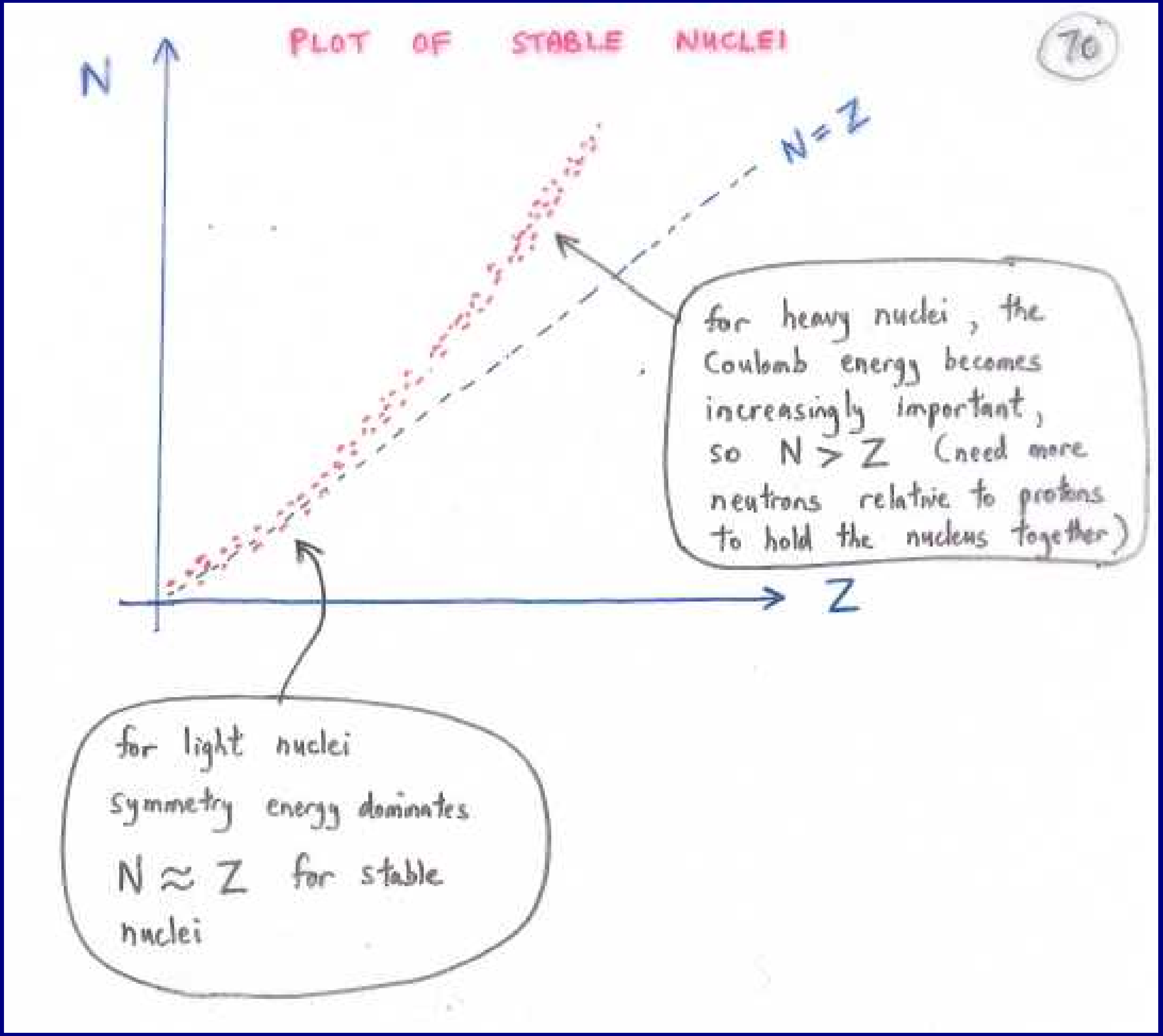
BINDING ENERGIES & EXPERIMENT

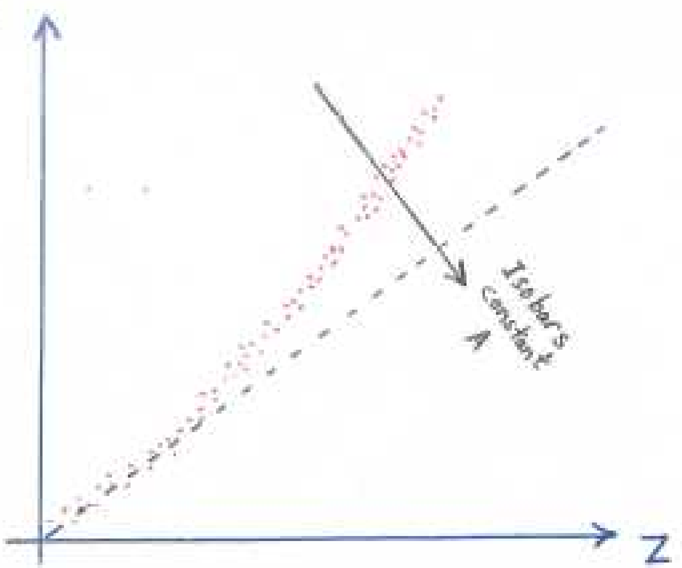


- Binding energy per nucleon of nuclei with even mass number A.
- Solid line corresponds to semi-empirical mass formula.
- Relatively large deviations for small A.
- For large A somewhat stronger binding at certain Z and N. These so-called 'magic numbers' will be discussed when we consider the shell model.



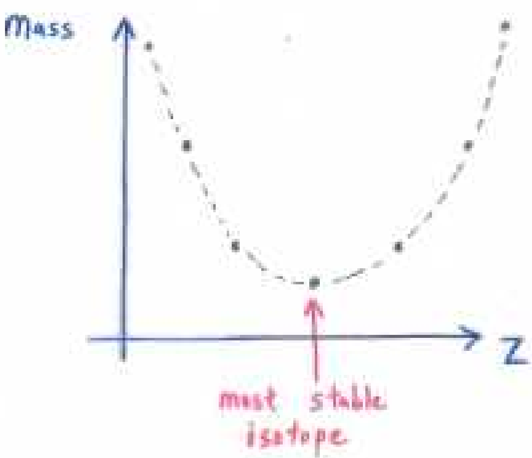
20
15
10
5
0
B/A [MeV]
Volume energy
Surface energy
Coulomb energy
Symmetry energy
Total binding energy
0
50
100
150
200
250
A


PLOT OF STABLE NUCLEI
70
N
N=Z
Z
for heavy nuclei, the Coulomb energy becomes increasingly important, so N > Z (need more neutrons relative to protons to hold the nucleus together)
for light nuclei symmetry energy dominates
N ≈ Z for stable nuclei

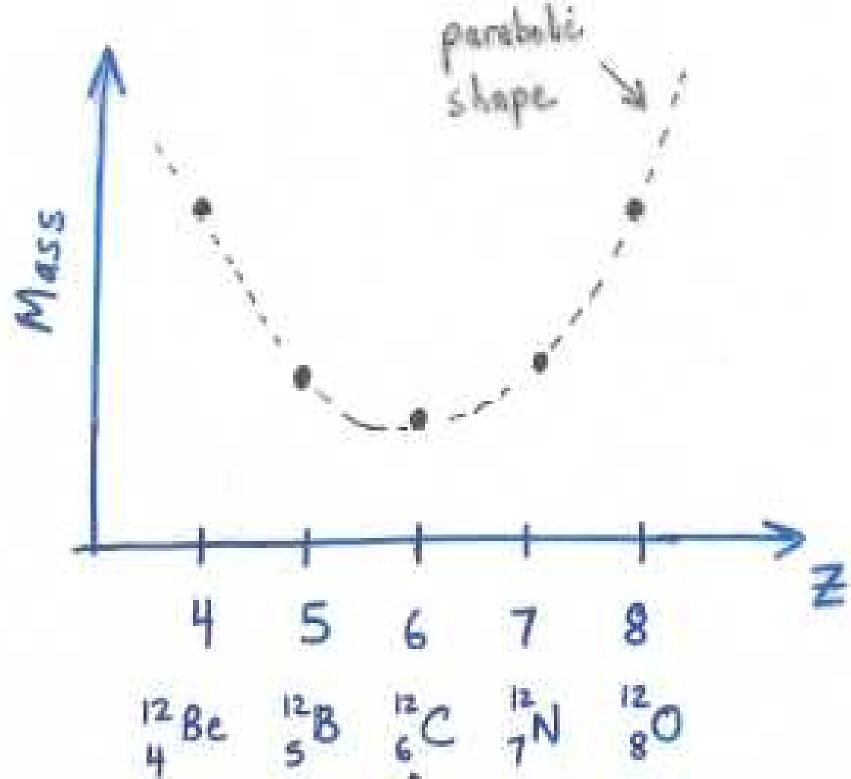


A = N + Z

If we move along a line of constant A, and plot the mass as a function of Z we get a parabolic shape like this



e.g. A = 12 isobars



mass smallest, hence binding greatest,

when $N = Z = 6$

MASS PARABOLA

- Let us consider nuclei with equal mass number A (isobars). The Weizsäcker-formula can be transformed into

$$M(A,Z) = \alpha A - \beta Z + \gamma Z^2 + \frac{\delta}{A^{1/2}}$$

where the coefficients are

$$\begin{aligned} \alpha &= m_n - a_v + a_s A^{-1/3} + \frac{a_a}{4} \\ \beta &= a_a + (m_n - m_p - m_e) \\ \gamma &= \frac{a_a}{A} + \frac{a_c}{A^{1/3}} \\ \delta & \quad \text{as before (-11.2 or 0 or +11.2 MeV)} \end{aligned}$$

- A plot of nuclear masses as a function of Z for constant A yields a mass parabola for odd A. For even A, the masses of the even-even and the odd-odd nuclei are found on two vertically shifted parabolas (by $2\delta/\sqrt{A}$)
- The minimum of the parabolas is found at $Z = \beta/2\gamma$. The nucleus with the smallest mass in an isobaric spectrum is stable with respect to β-decay.

What happens when you have a nucleus off the bottom of the valley of stability?

The nucleus can "roll towards the bottom of the valley" by changing protons to neutrons (or neutrons to protons), keeping A constant.

Recall

$$n \longrightarrow p + e^- + \bar{\nu}_e$$

β^- decay
free neutron or in a nucleus

$$p \longrightarrow n + e^+ + \nu_e$$

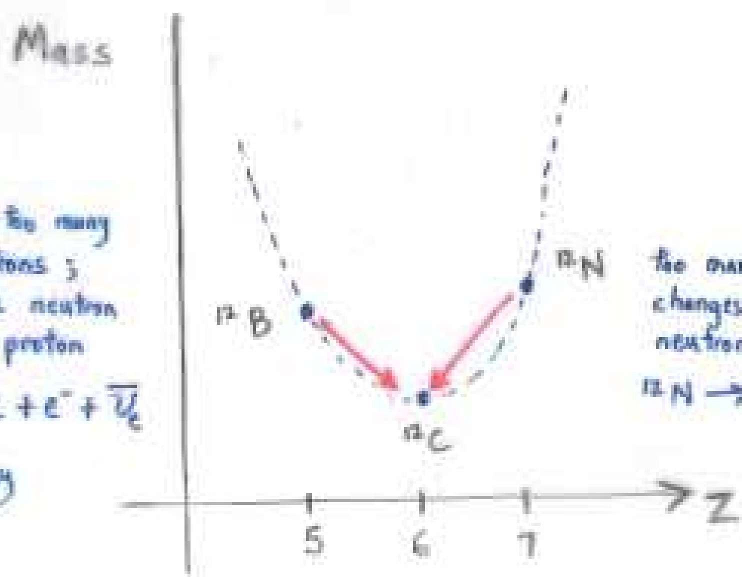
β^+ decay
only in a nucleus

Example: mass 12 system ($A = 12$)

^{12}B has too many neutrons; changes a neutron to a proton

$$^{12}B \rightarrow {}^{12}C + e^- + \bar{\nu}_e$$

β^- decay

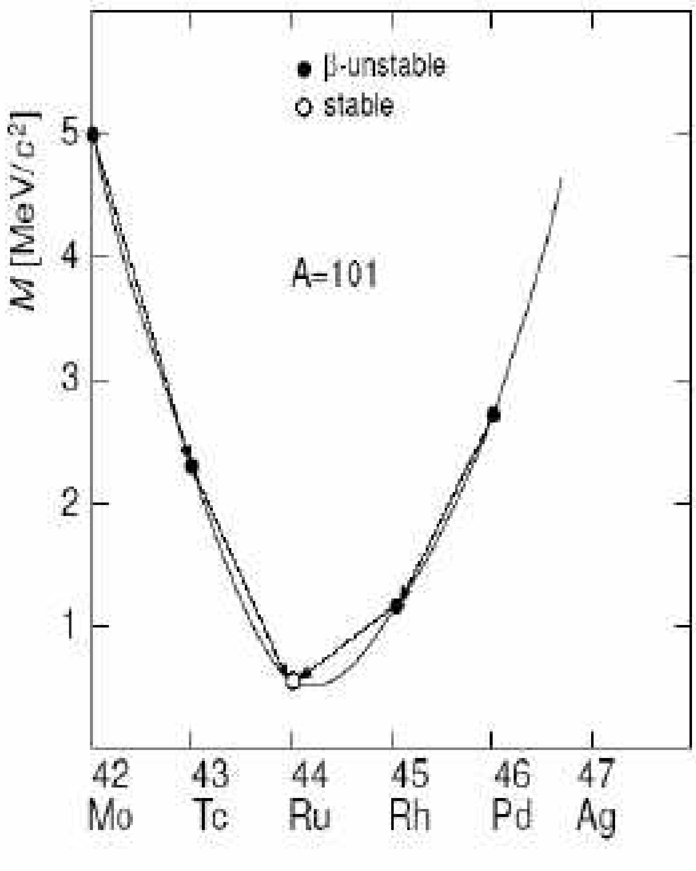


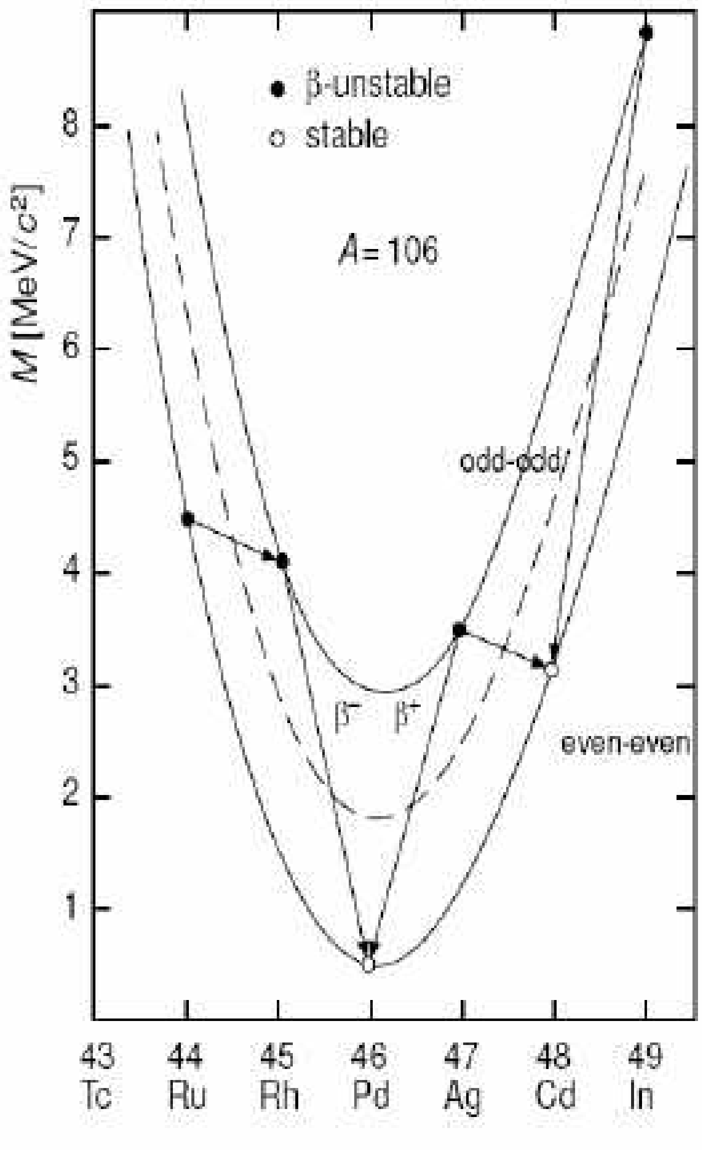
too many protons; changes a proton to a neutron

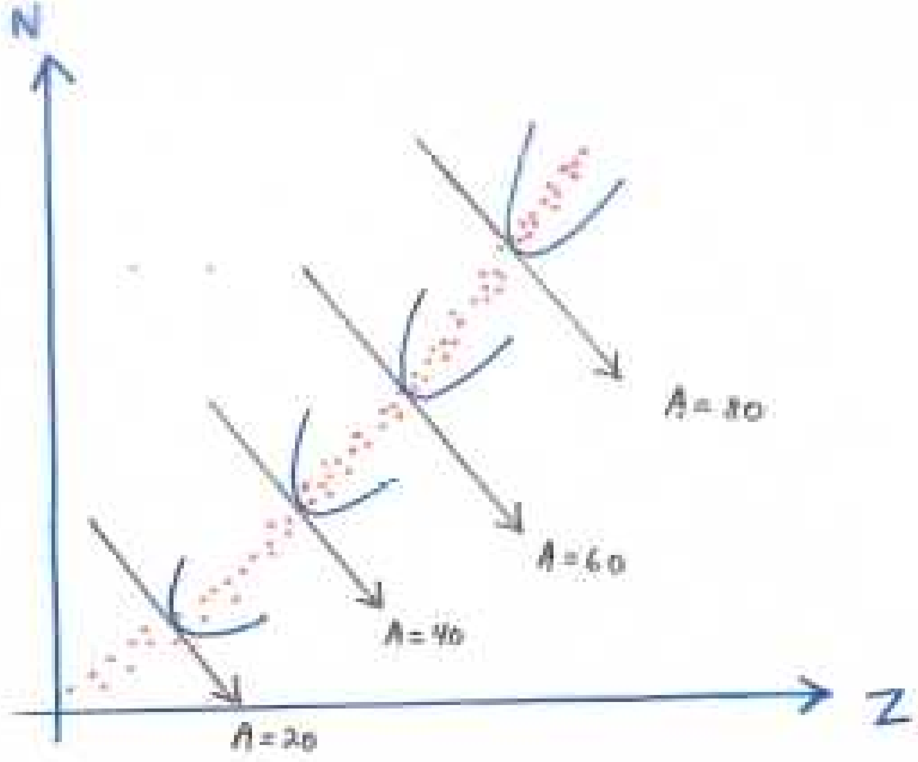
$$^{12}N \rightarrow {}^{12}C + e^+ + \nu_e$$

β^+ decay

MASS PARABOLA







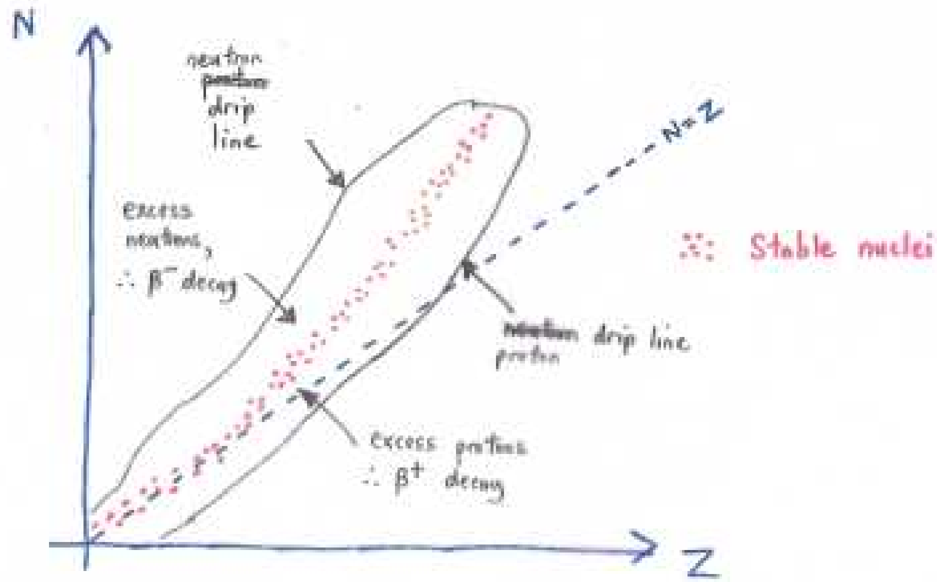
If we now repeat this procedure for many different values of A, and plot the mass in the third dimension (out of the page), the mass parabolas form a valley or canyon in the N-Z plane. This is called the "valley of stability".

Particle instability

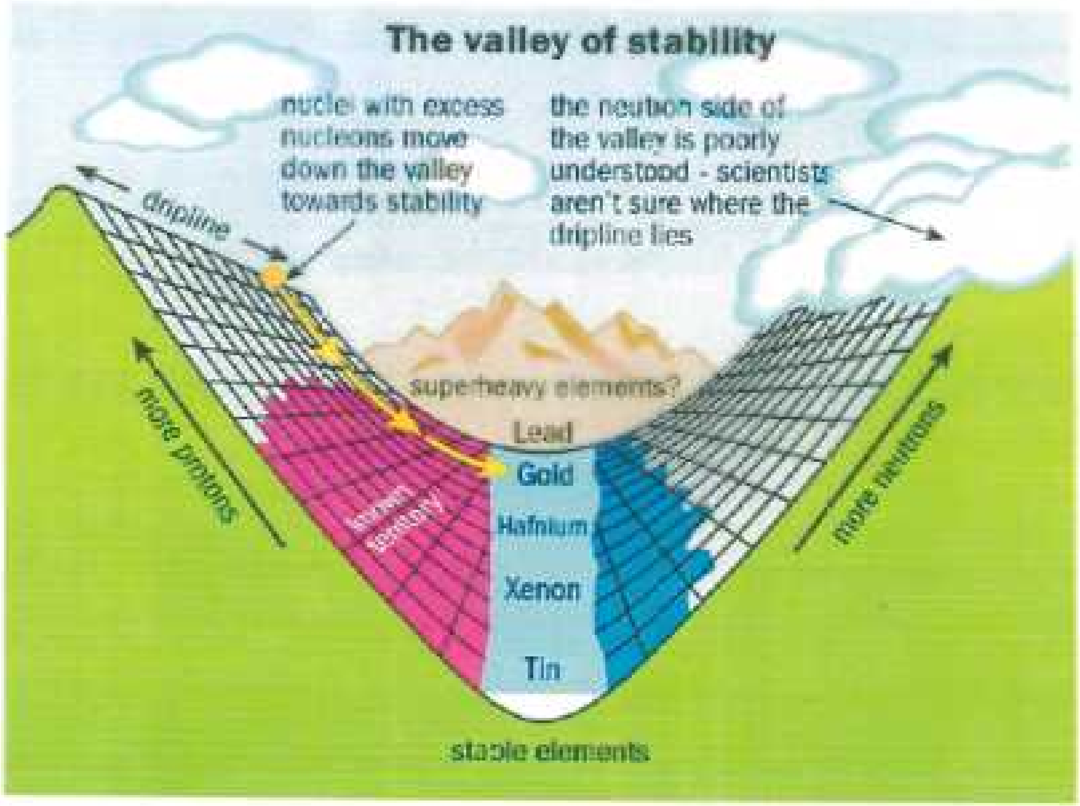
as we climb further and further away from the valley of stability, the nuclei become more and more unstable, and decay by $\beta^\pm$ decay. Eventually we reach a point where the nucleus is so unstable that it cannot remain bound — it falls apart as soon as it is made.

At this point, the proton or neutron separation energy becomes negative — it takes no energy at all to remove a proton or neutron.

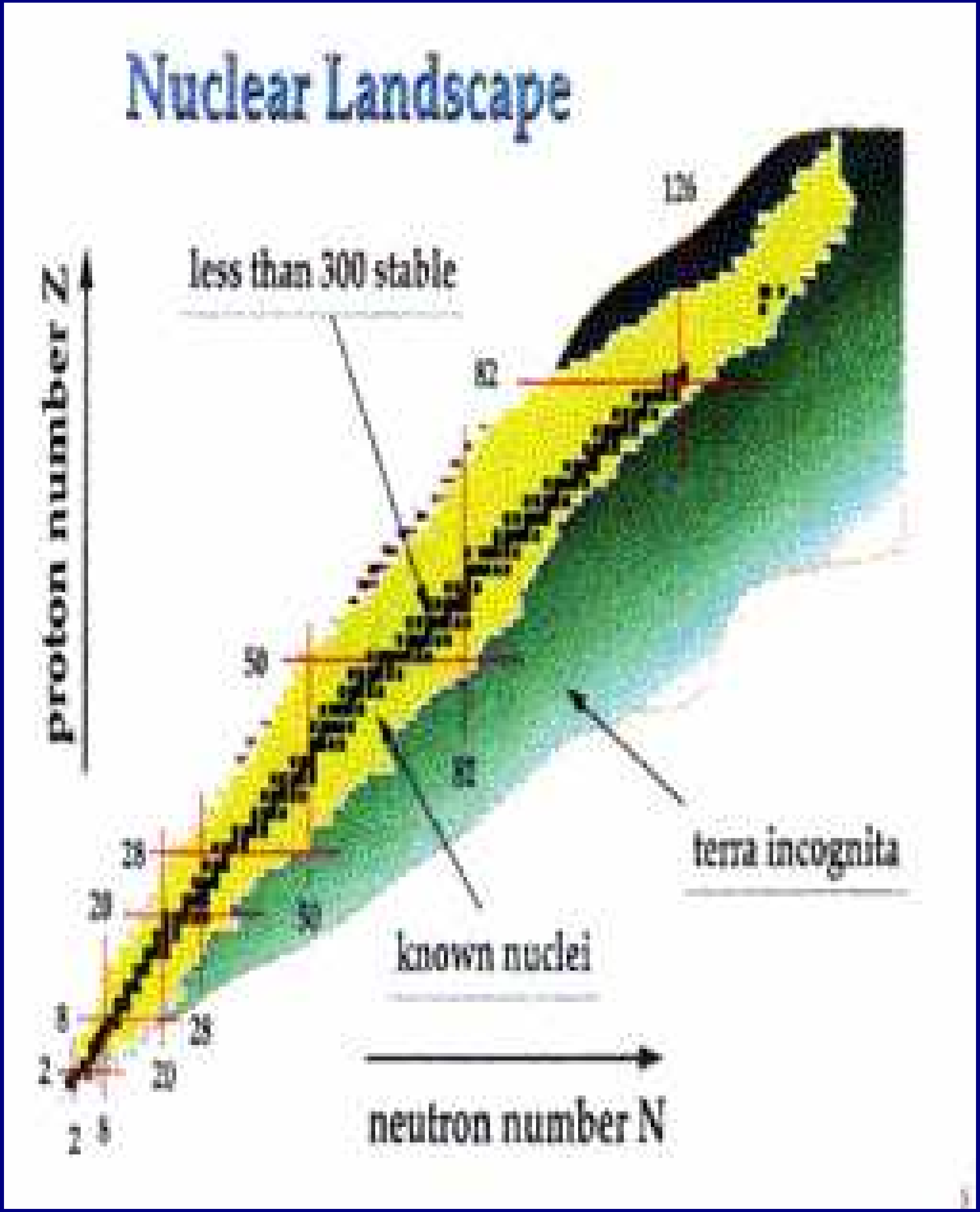
These are called the drip lines on the N-Z plot, because nuclei there would be dripping their excess protons or neutrons.



The valley of stability



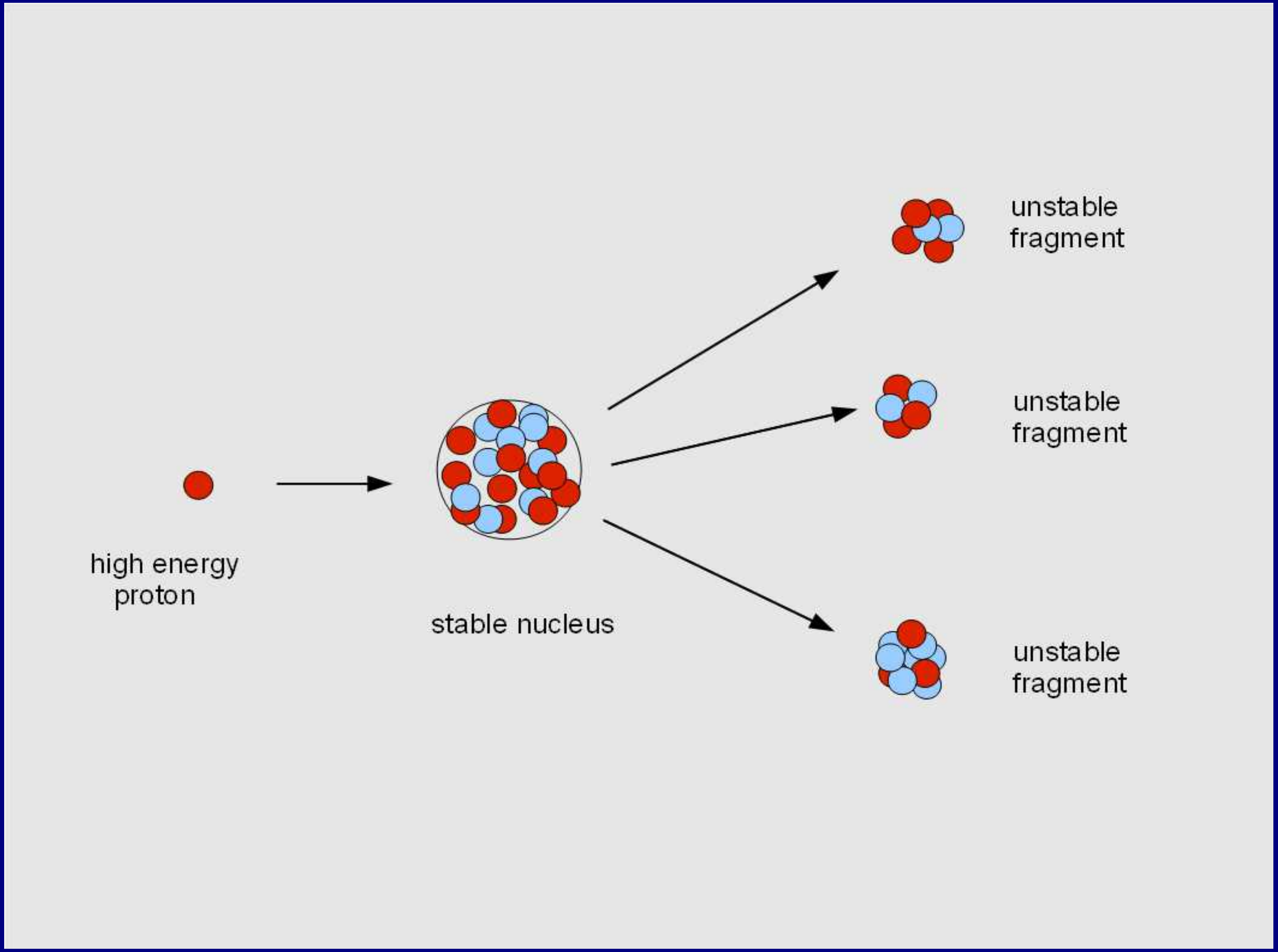
The "valley of stability" - new nuclear machines such as the Rare Isotope Accelerator will open up studies of nuclear phenomena using beams of short-lived isotopes, which form the high "walls" of the valley.

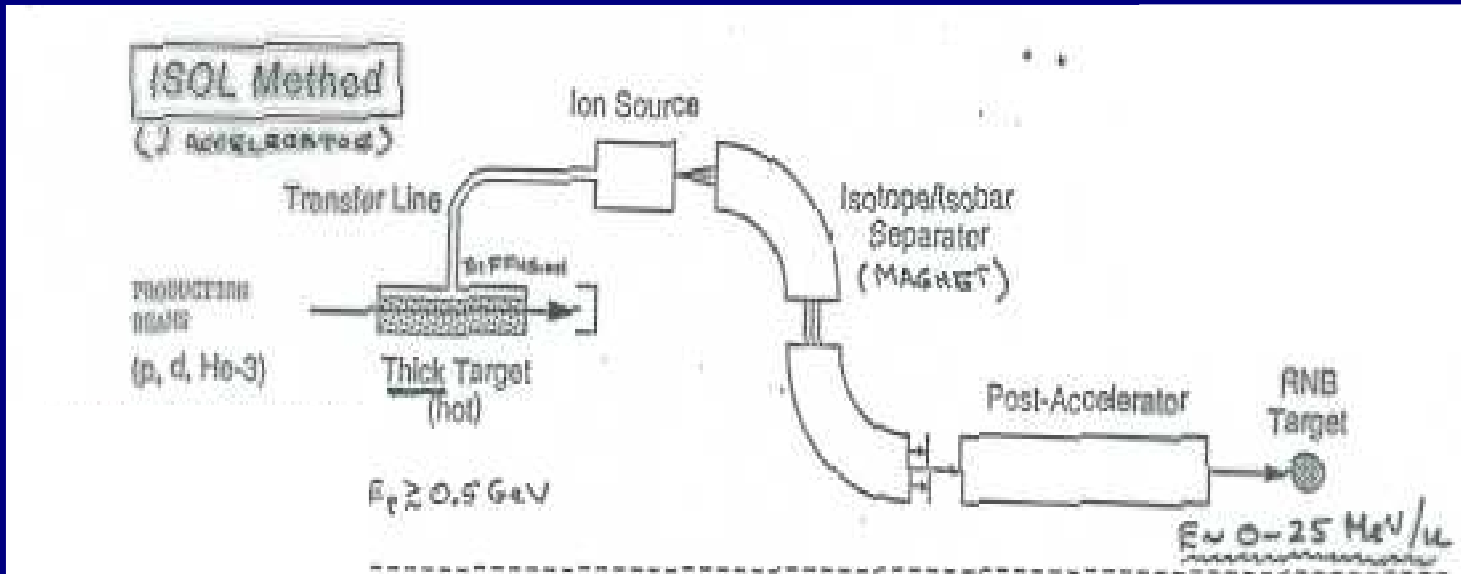


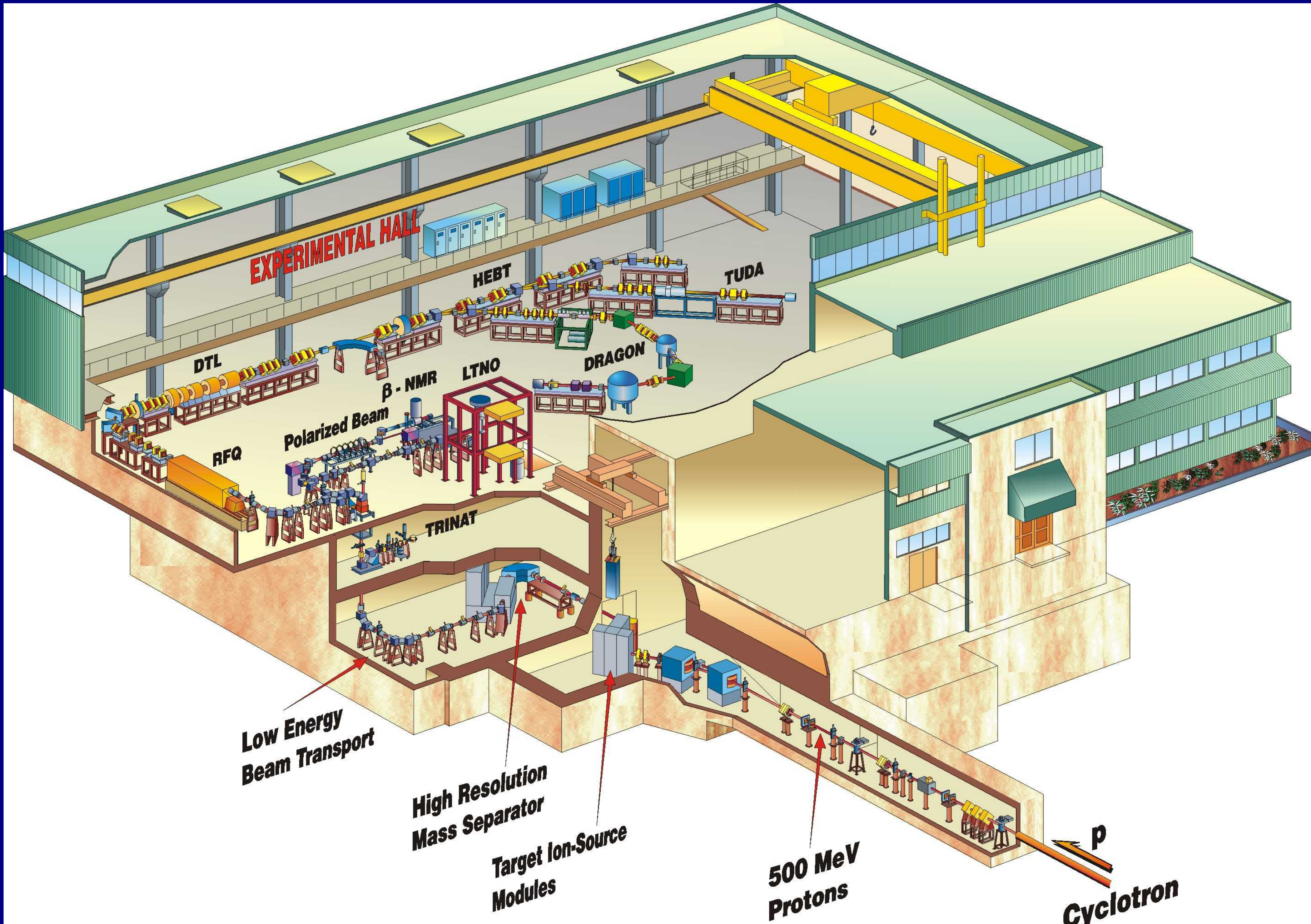
Purpose of ISAC and other radiioactive ion-beam labs is to study the properties of nuclei far from the valley of stability – their masses their structure, the reactions that they undergo, novel modes of nuclear excited states, etc. etc.

Even the exact limits of nuclear stability are not well established.

Nuclei far from the valley of stability by means of a **spallation** reaction on a heavy stable nucleus




ISOL Method
Ion Source
Transfer Line
Isotope/Isobar
Separator
(MAGNET)
(p, d, He-3)
Thick Target
(hot)
Post-Accelerator
RNB
Target
$E_t \gtrsim 0.5$ GeV
E ~ 0-25 MeV/u


EXPERIMENTAL HALL
HEBT
TUDA
DTL
DRAGON
LTNO
β - NMR
Polarized Beam
RFQ
TRINAT
Low Energy
Beam Transport
High Resolution
Mass Separator
Target Ion-Source
Modules
500 MeV
Protons
p
Cyclotron

In summary:

1. The mass and binding energy of a nucleus is an interplay between the attractive strong interaction and the repulsive Coulomb interaction.

2. The liquid drop model provides a simple way to understand the systematic features of nuclear masses and binding energies of nuclei close to the valley of stability

3. ISAC and other labs like it explore the properties of nuclei far from the valley of stability.