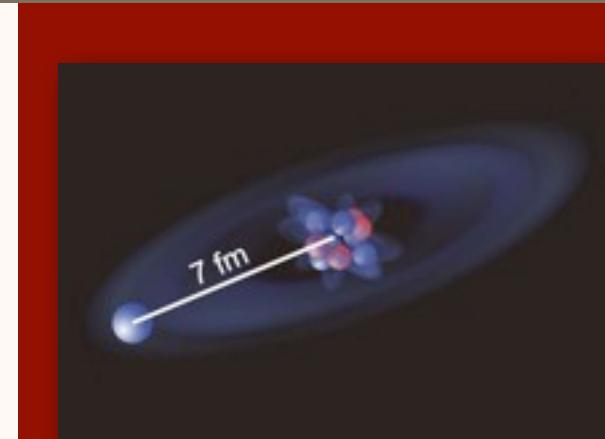


Calculations for Halo Nuclei: the ${}^6\text{He}$ case

Sonia Bacca | Theory Group | TRIUMF

Theory/ISAC meeting
May 18 2012



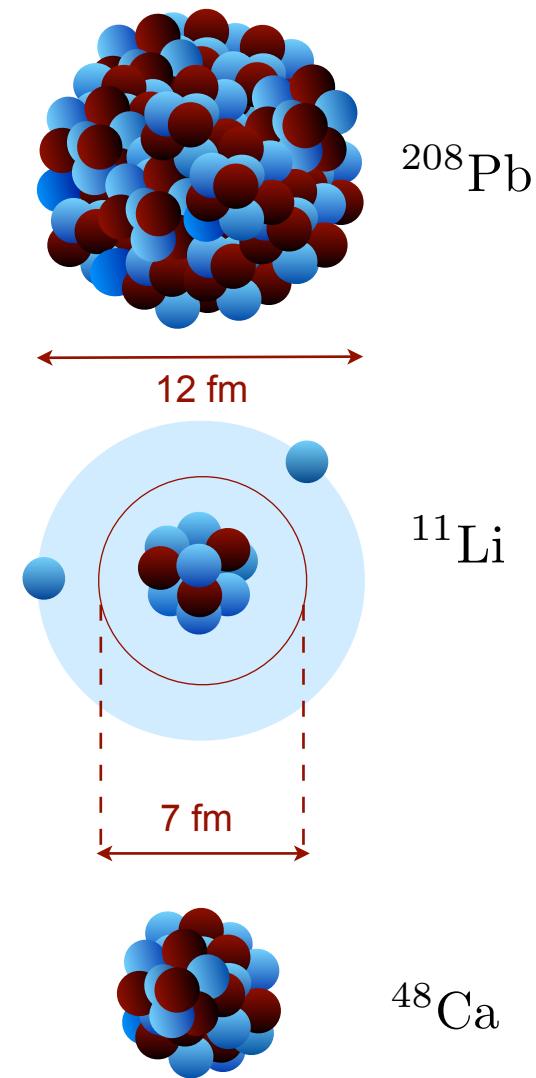
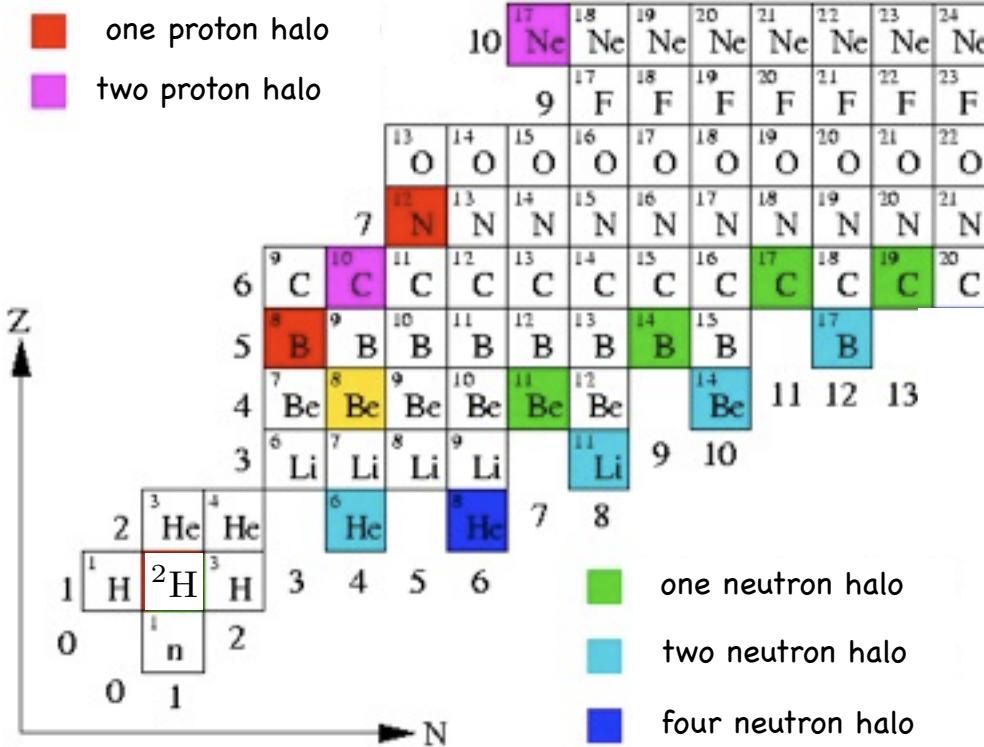
Nuclear Halo



Moon Halo

Owned and operated as a joint venture by a consortium of Canadian universities via a contribution through the National Research Council Canada
Propriété d'un consortium d'universités canadiennes, géré en co-entreprise à partir d'une contribution administrée par le Conseil national de recherches Canada

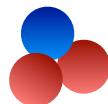
Halo Nuclei



The Helium Isotope Chain

Shows many interesting features:

^3He



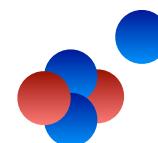
bound

^4He



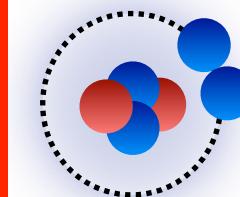
bound

^5He



unbound

^6He



bound

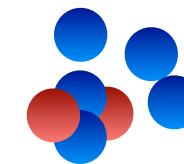
halo

Borromean system



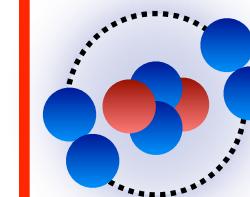
lives 806 ms

^7He



unbound

^8He



bound

halo

Most exotic nucleus
“on earth”

$$\frac{N}{Z} = 3$$

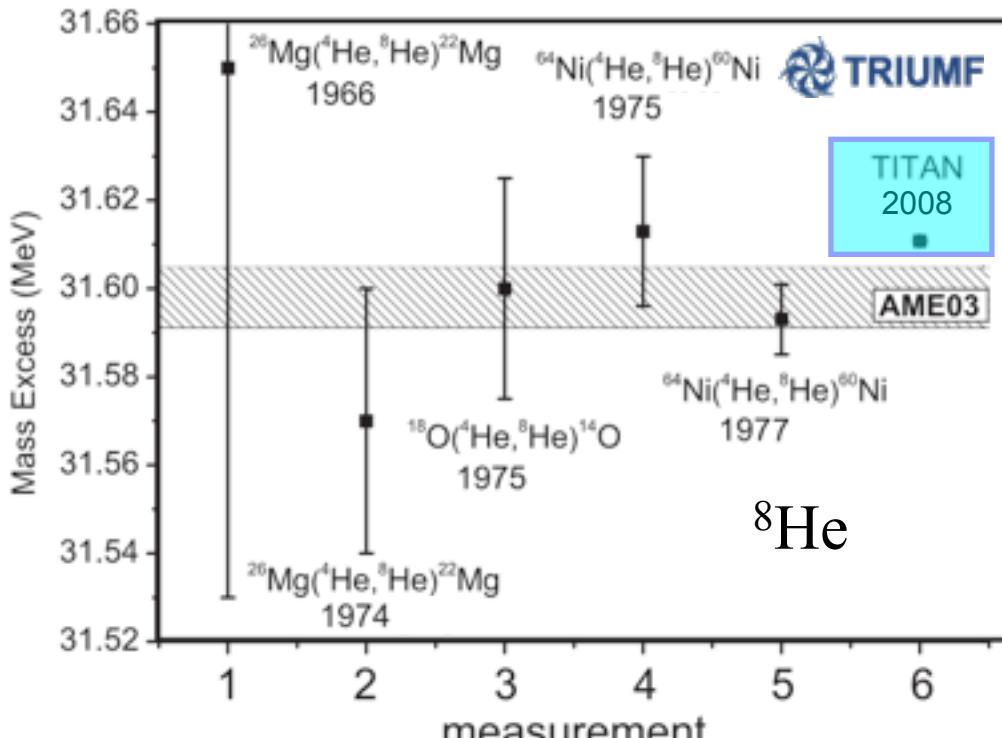
lives 108 ms

Even if they are exotic short lived nuclei, they can be investigated experimentally. From a comparison of theoretical predictions with experiment we can test our knowledge on nuclear forces in the neutron rich region

Helium Halo Nuclei - Experiment

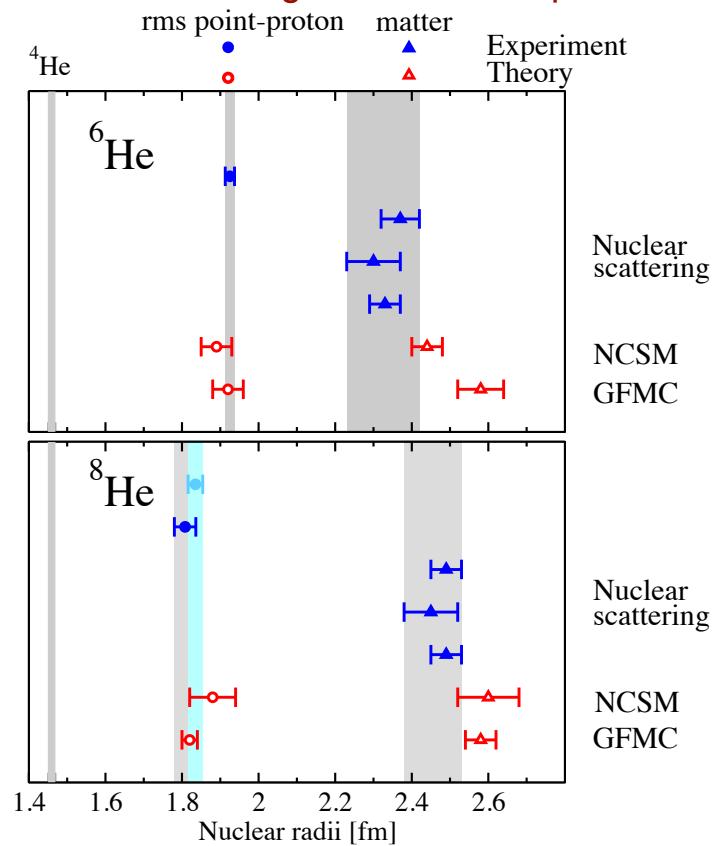
New Era of Precision Measurements for masses and radii

Mass measurement of ^8He with the Penning trap



TRIUMF, Ryjkov et al. PRL 101, 012501 (2008)

Measurement of charge radii via isotope shift



ARGONNE, Wang et al. PRL 93, 142501 (2004)
GANIL, Mueller et al. PRL 99, 252501 (2007)

$$\delta\nu_{AA'} = \delta\nu_{A,A'}^{mass} + K\delta\langle r_{ch}^2 \rangle_{AA'}$$

$$\langle r_p^2 \rangle = \langle r_{ch}^2 \rangle - \langle R_p^2 \rangle - \frac{3}{4M_p^2} - \frac{N}{Z} \langle R_n^2 \rangle$$

Masses and radii of helium isotopes
are important challenges for theory!

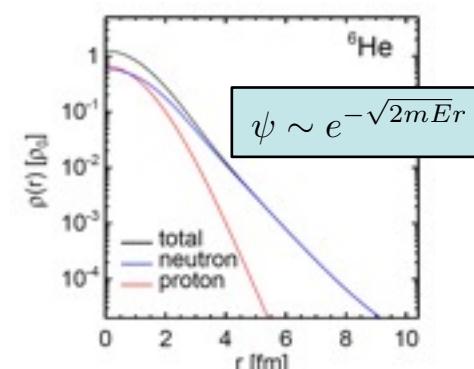
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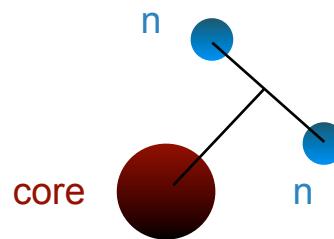
Helium Halo Nuclei -Theory

Why are halo nuclei a challenge to theory?

- It is difficult to describe the long extended wave function
- They test nuclear forces at the extremes, where less is known



Cluster models:



3-body models with phenomenological interactions

${}^6\text{He}$, ${}^{11}\text{Li}$ - borromean systems

can do reactions, Faddeev calculations

but difficult to add core polarizations



Efros, Fedorov,
Garrido, Hagino,
Bertulani, ...

New: Revived by halo EFT

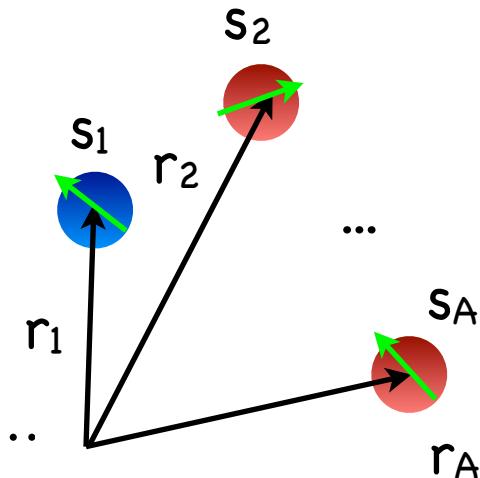
Bertulani, Hammer, Higa, Philips, Ji ...

Ab -initio calculations

- Start from neutrons and protons as building blocks
(relative coordinates, spins, isospins)
- Solve the non-relativistic quantum mechanical problem of A-interacting nucleons

$$H|\psi_i\rangle = E_i|\psi_i\rangle$$

$$H = T + V_{NN} + V_{3N} + \dots$$



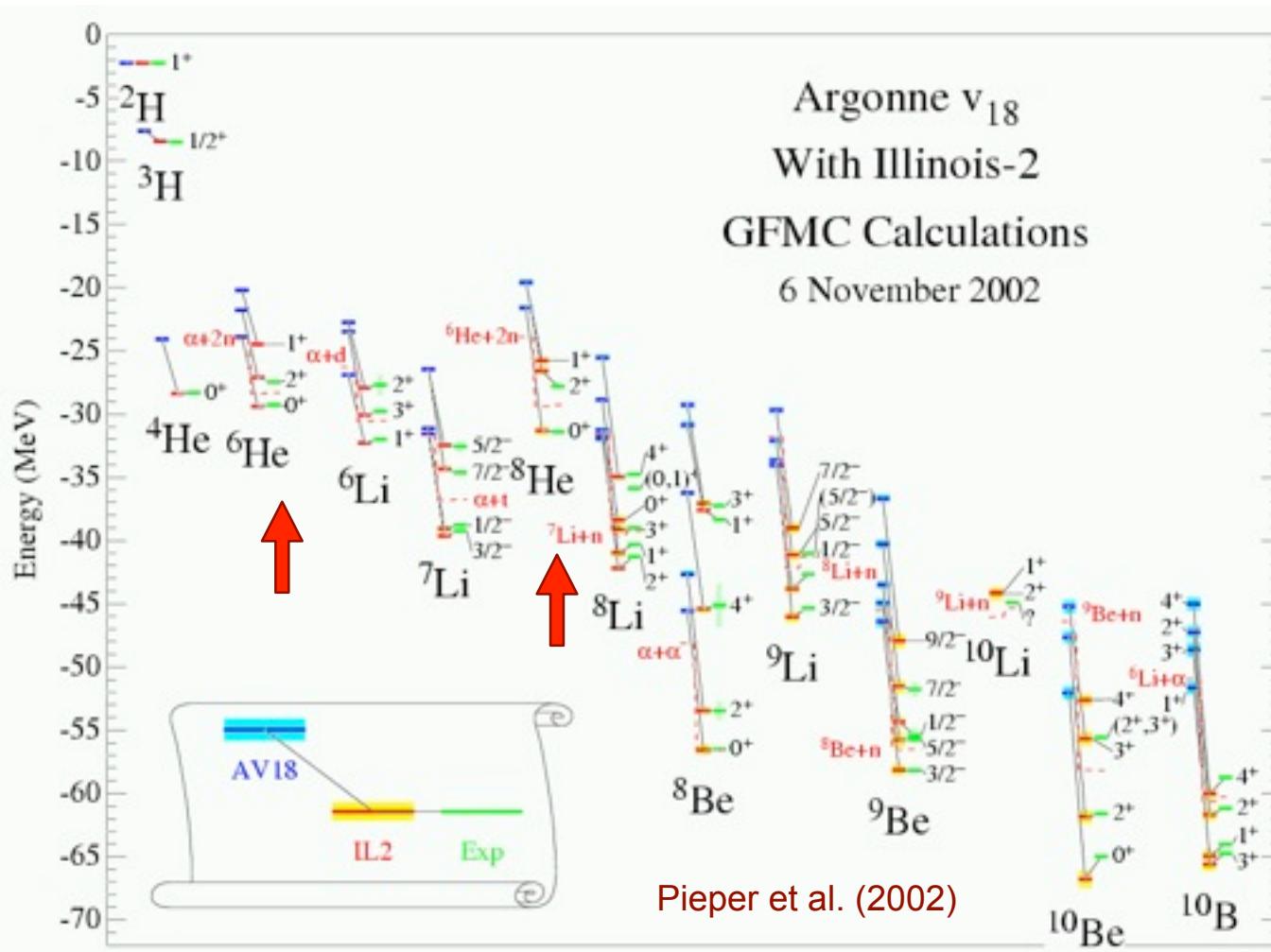
- Find numerical solutions with no approximations or controllable approximations
- Calculate low-energy observables and compare with experiment to **test nuclear forces** and investigate the role of many-nucleon forces



Green Function Monte Carlo

Quantum Monte Carlo Method,
Uses local two- and three-nucleon forces

→ short range phenomenology



AV18

does not bind the helium halo
with respect to 2n emission

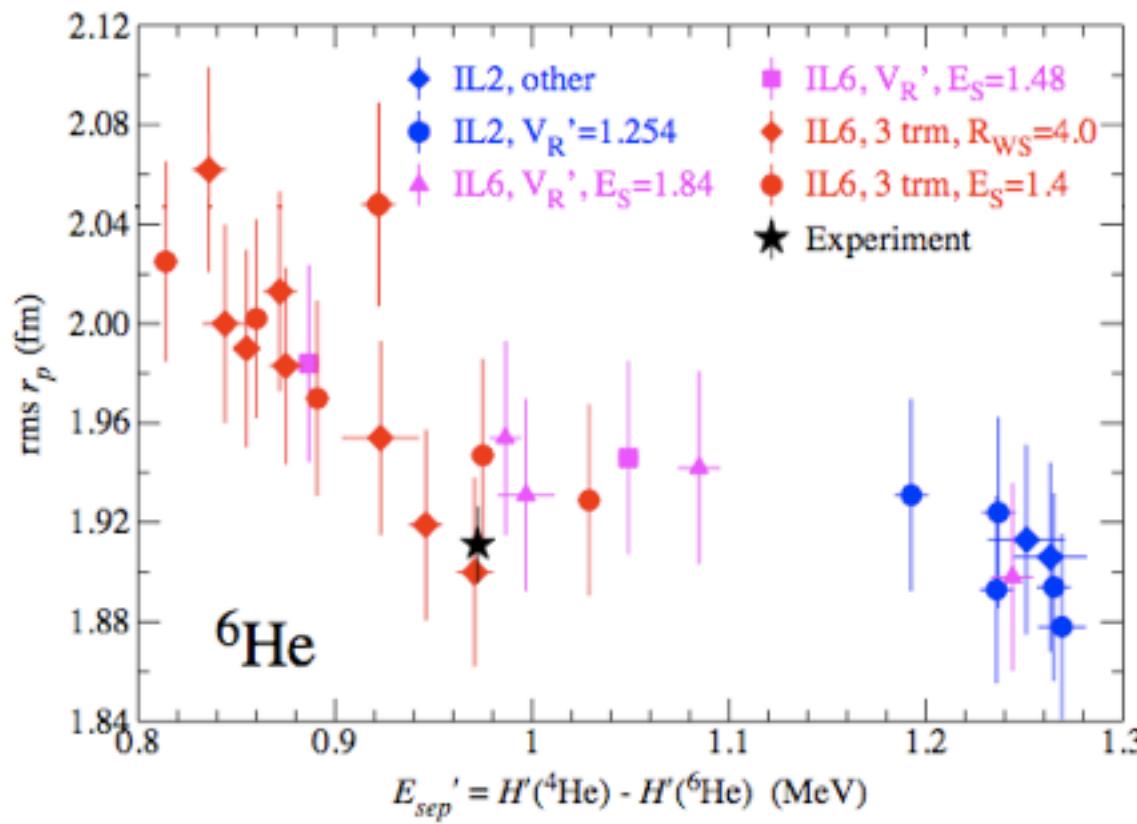
IL2

$$V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^{3\pi,R} + V_{ijk}^R$$

N.B.: parameters of the IL2 force are obtained from a fit of 17 states of $A < 9$ including the binding energy of ${}^6\text{He}$ and ${}^8\text{He}$

Green Function Monte Carlo

- Estimation of the proton radius -



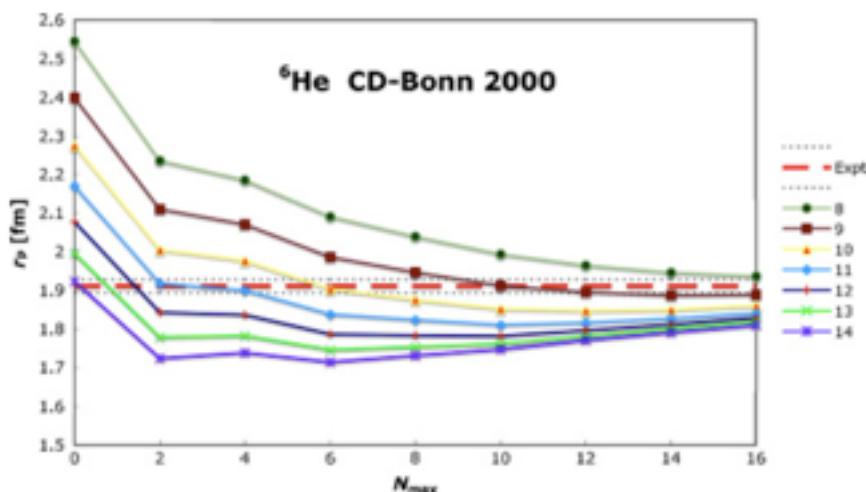
S.C. Pieper, arXiv:0711.1500, proceedings of Enrico Fermi School

NCSM

Diagonalization Method using Harmonic Oscillator Basis

$$\psi_{nl}(r) \sim e^{-\nu r^2} L_n^{l+1/2}(2\nu r^2) \quad \nu = m\omega/2\hbar$$

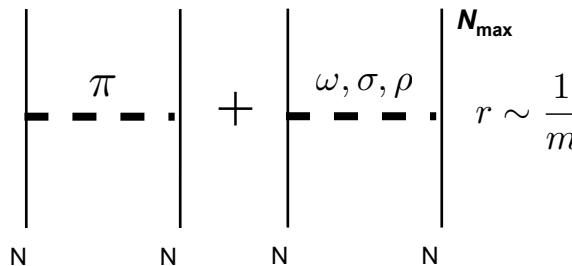
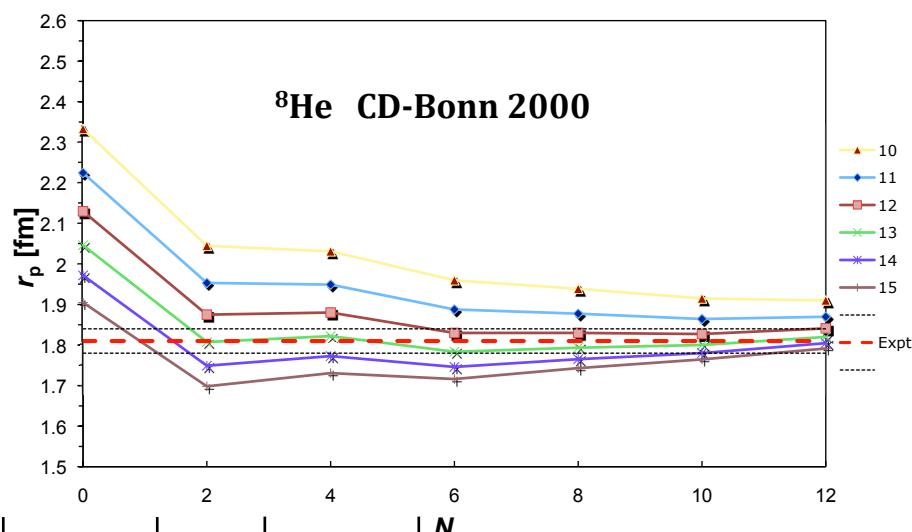
Caurier and Navratil, PRC 73, 021302(R) (2006)



HO parameter dependence in radius

CD-Bonn \rightarrow meson exchange theory

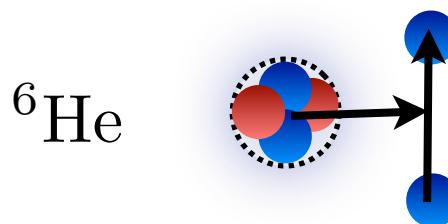
E_B [MeV]	Expt.	CD-Bonn 2000
${}^4\text{He}$	28.296	26.16 (6)
${}^6\text{He}$	29.269	26.9 (3)
${}^8\text{He}$	31.408 (7)	26.0 (4)



This potential underbinds helium isotopes

Other Calculations

Microscopic Cluster Model (MCM) Brida and Nunes, NPA 847,1 (2010)



${}^4\text{He}$ has a structure, but it is frozen in the ground state (SVM)

NN force is the Minnesota potential (no tensor force)

Fermionic Molecular Dynamics (FMD) Neff and Feldmeier, NPA 738,357 (2004)

Slater Determinant $|Q\rangle = \mathcal{A}(|q_1\rangle \otimes \dots \otimes |q_A\rangle)$

Single particle Hilbert space $\langle \mathbf{x}|q\rangle = \sum_i c_i \exp\left\{-\frac{(\mathbf{x} - \mathbf{b}_i)^2}{2a_i}\right\} \otimes |\chi_i^\uparrow, \chi_i^\downarrow\rangle \otimes |\xi\rangle$

Gaussian wave-packets localized in phase-space,
spin free, isospin fixed

Single particle complex variational parameters $b_i^x, b_i^y, b_i^z, a_i \chi_i^\uparrow, \chi_i^\downarrow$ Go up to $A \approx 60$ and can do halo

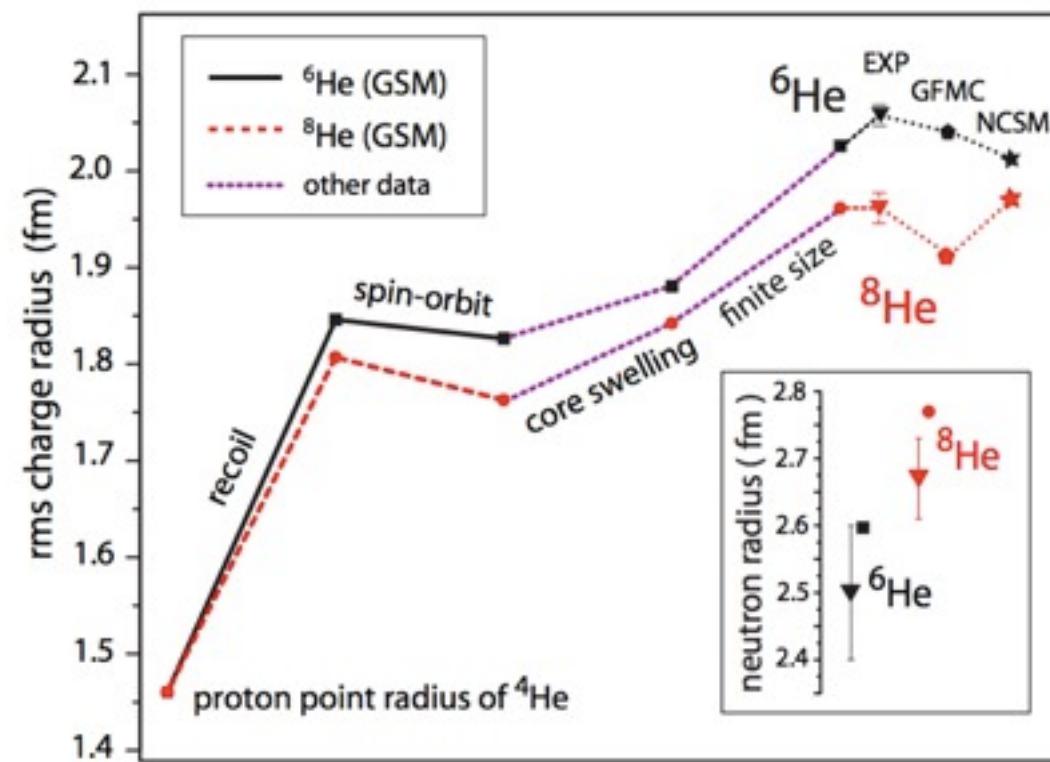
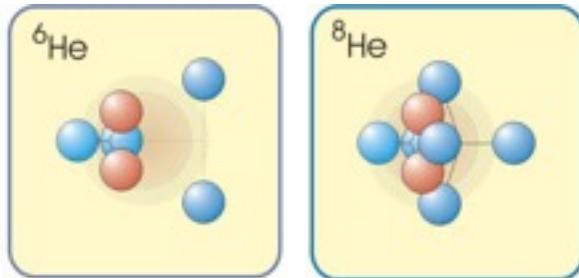
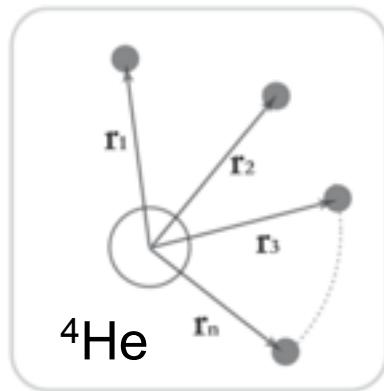
Multiconfiguration Calculations $|Q\rangle = \sum |Q^a\rangle$ diagonalize a (small) matrix

Hamiltonian $H = T + V_{\text{UCOM}} + \delta V_{c+p+ls}^a$

Other Calculations (more phenomenological)

Gamow Shell Model Papadimitriou *et al.*, PRC 84, 051304 (2011)

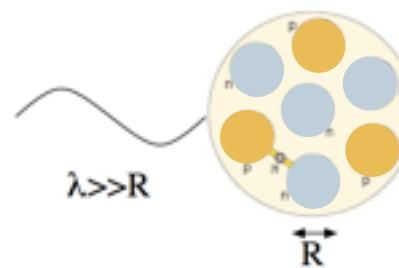
Assuming a core in the calculations and using phenomenological potentials



Nuclear Forces from EFT

The Effective Field Theory approach

$$\frac{1}{\lambda} = Q \ll \Lambda_b = \frac{1}{R}$$



Limited resolution at low energy
expand in powers

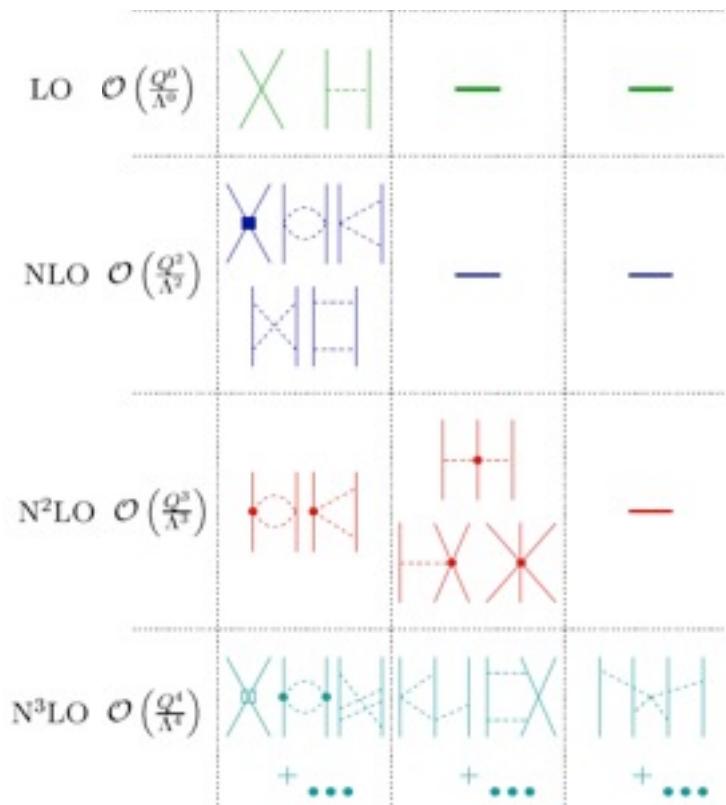
$$\mathcal{L} = \sum_k c_k \left(\frac{Q}{\Lambda_b} \right)^k$$

Details of short distance physics not resolved,
but captured in short range couplings, fit to
experiment once

Systematic and can provide error estimates

$$H(\Lambda) = T + V_{NN}(\Lambda) + V_{3N}(\Lambda) + V_{4N}(\Lambda) + \dots$$

NN	3N	4N
----	----	----



$$V_{NN} > V_{3N} > V_{4N}$$

Weinberg, van Kolck, Kaplan, Savage, Weise,
Epelbaum, Meissner, Nogga, Machleidt,...

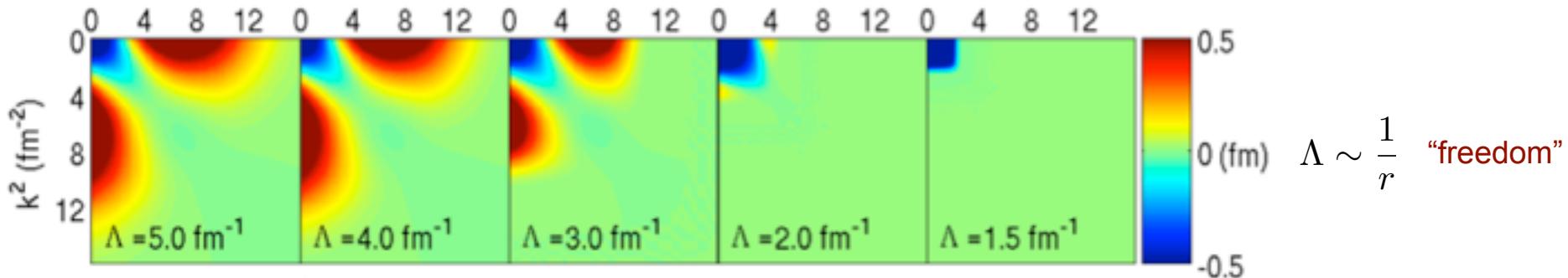
Low-momentum Forces from EFT

Effective field theory potentials and low-momentum evolution

Evolution of 2N forces: phase-shift equivalent

Low-momentum interactions: Bogner, Kuo, Schwenk (2003) need smaller basis

Like acting with a unitary transformation $U^{-1}VU$ still preserve phase-shifts and properties of 2N systems

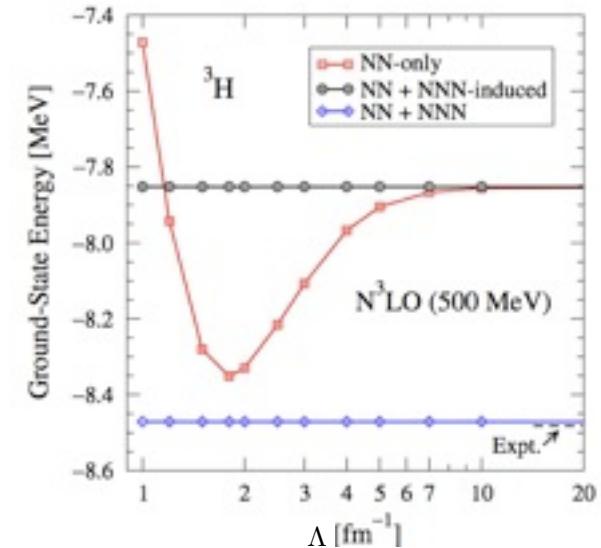


$$H(\Lambda) = T + V_{NN}(\Lambda) + V_{3N}(\Lambda) + \dots$$



Variation of the cutoff provides a tool to estimate the effect of 3N forces

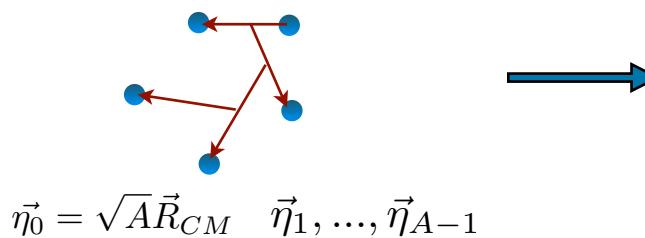
Can evolve consistently 3N forces:
Jurgenson, Navratil, Furnstahl, (2009)



Hyper-spherical harmonics

- Few-body method - uses relative coordinates

$$|\psi(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_A)\rangle = |\varphi(\vec{R}_{CM})\Psi(\vec{\eta}_1, \vec{\eta}_2, \dots, \vec{\eta}_{A-1})\rangle$$



Recursive definition of hyper-spherical coordinates

$$\rho, \Omega \quad \rho^2 = \sum_{i=1}^A r_i^2 = \sum_{i=1}^{A-1} \eta_i^2$$

$$H(\rho, \Omega) = T_\rho + \frac{K^2(\Omega)}{\rho^2}$$

$$\Psi = \sum_{[K], \nu}^{K_{max}, \nu_{max}} c_\nu^{[K]} e^{-\rho/2} b^n L_\nu^n(\frac{\rho}{b}) [\mathcal{Y}_{[K]}^\mu(\Omega) \chi_{ST}^{\bar{\mu}}]_{JT}^a$$

K_{max}, ν_{max}

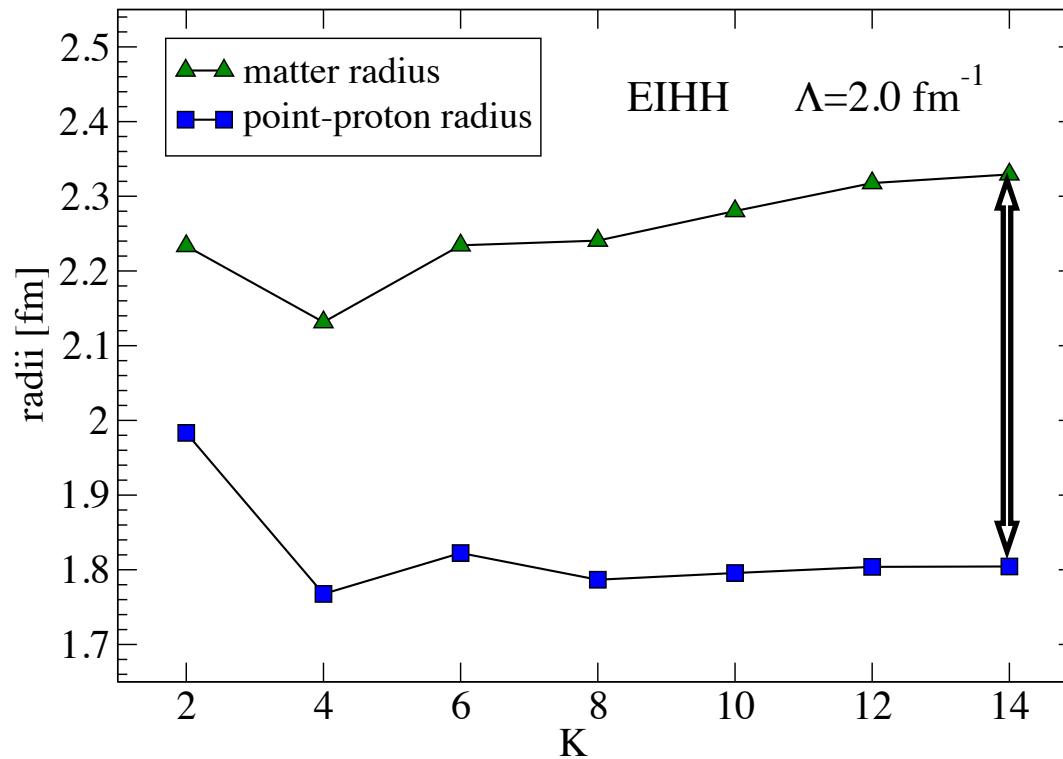


Asymptotic $e^{-a\rho}$ $\rho \rightarrow \infty$

To solve Schrödinger equation \rightarrow diagonalize a big matrix

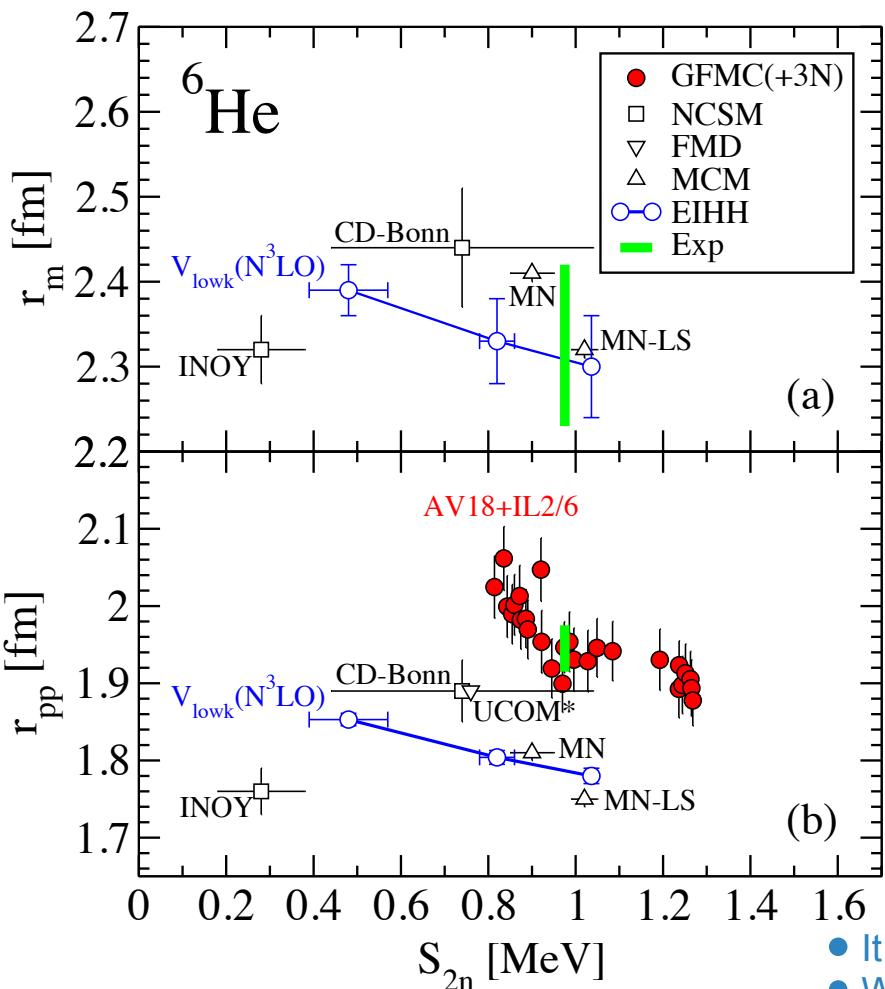
^6He from hyper-spherical harmonics

Signatures of the halo



- Excellent convergence of r_{pp}
- Matter radius converge also, but slower

Comparison with experiment



M. Brodeur *et al.* Phys. Rev. Lett. 108, 052504 (2012)
& S.B.. *et al.* arXiv:1202.0516

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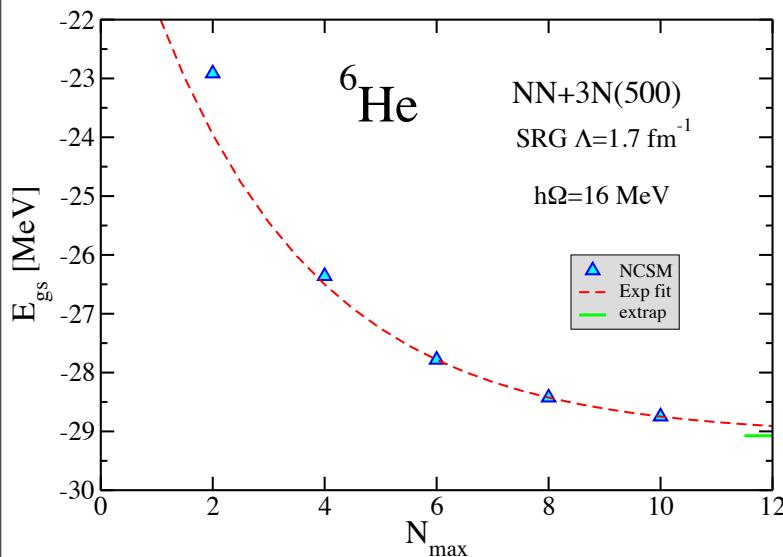
- It is important to compare more than one observable together
- We observe a correlation between radii and separation energy
- Theory needs (improved) 3NFs

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Future:
Include 3NF

NCSM-Three-nucleon forces

Petr Navratil (TRIUMF), Robert Roth (TUD), et al.



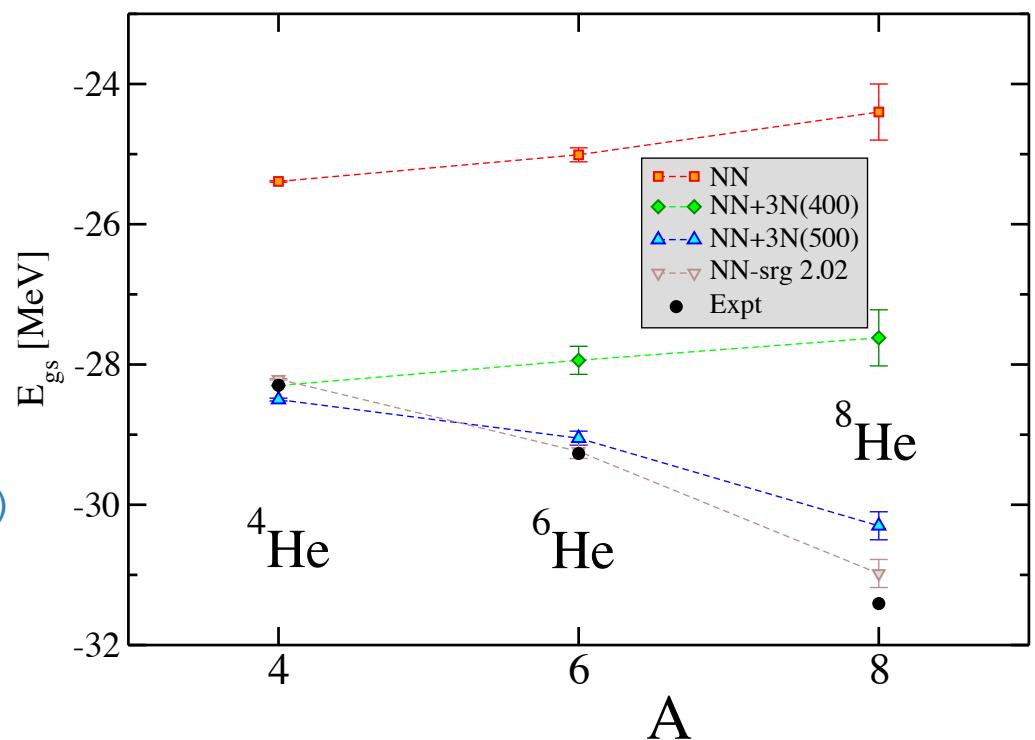
- Low-momentum NN forces only give similar results as chiral NN+3NF(500)

$$\frac{c_E}{F_\pi^4 \Lambda_b} \vec{\tau}_1 \cdot \vec{\tau}_2 \delta(\vec{r}_1 - \vec{r}_2) \delta(\vec{r}_3 - \vec{r}_1) =$$

$$\frac{c_E}{F_\pi^4 \Lambda_b} \vec{\tau}_1 \cdot \vec{\tau}_2 \int d\pi_1 d\pi_2 d\pi'_1 d\pi'_2 |\pi_1 \pi_2\rangle \langle \pi'_1 \pi'_2|$$

$F(Q^2, \Lambda) = \exp(-Q^4/\Lambda^4)$

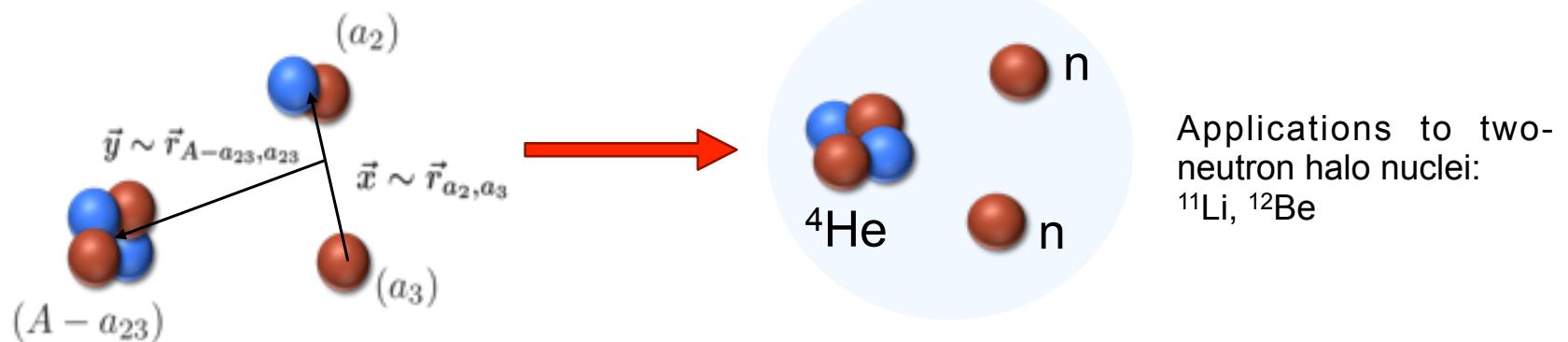
c_E



NCSM/RGM for a three-body cluster

Carolina Romero-Redondo (TRIUMF), Petr Navratil (TRIUMF), Sofia Quaglioni (LLNL)

There are several nuclear systems that have a three-body cluster configuration



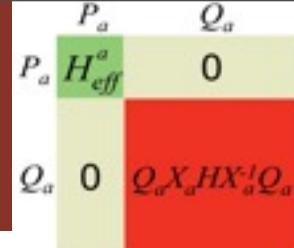
Three-clusters: internal NCSM wave function of each of the three clusters in the system.

Reaction and structure problems that can be studied with the NCSM/RGM with three-body cluster states

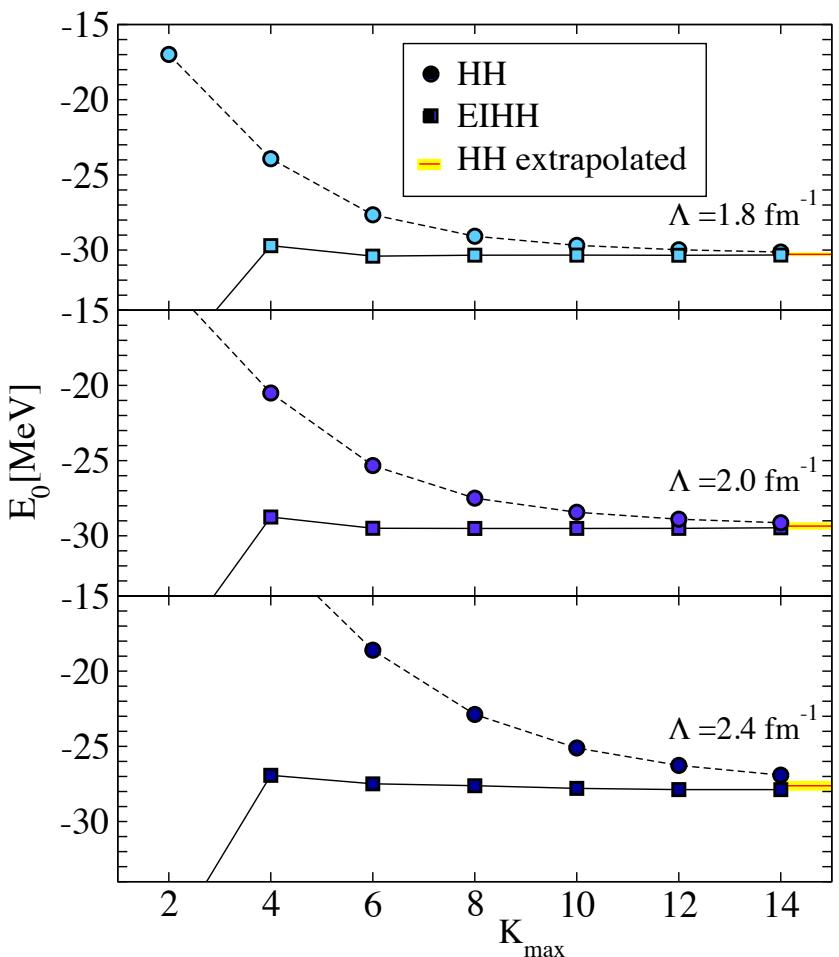
Solve an integral-differential Schrödinger-like equations once you have calculated the kernels

$$\sum_{\nu} \int dx dy x^2 y^2 [\mathcal{H}_{\nu' \nu}(x, y, x', y') - E \mathcal{N}_{\nu' \nu}(x, y, x', y')] G_{\nu}^{J^\pi T}(x, y) = 0$$

^6He from hyper-spherical harmonics



Interaction: $V_{\text{low } k}$ from N³LO (500 MeV)



- EIHH agrees with extrapolated HH results from EPJ A 42, 553 (2009)

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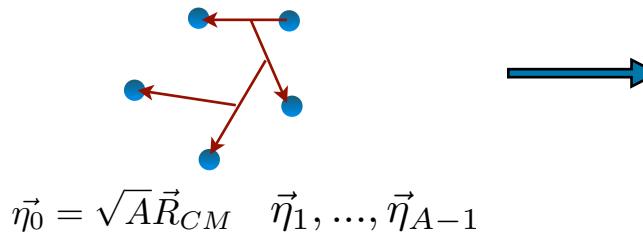
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- EI is key to reach a reliable convergence of radii

Hyper-spherical harmonics

- Few-body method - uses relative coordinates

$$|\psi(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_A)\rangle = |\varphi(\vec{R}_{CM})\Psi(\vec{\eta}_1, \vec{\eta}_2, \dots, \vec{\eta}_{A-1})\rangle$$



Recursive definition of hyper-spherical coordinates

$$\rho, \Omega \quad \rho^2 = \sum_{i=1}^A r_i^2 = \sum_{i=1}^{A-1} \eta_i^2$$

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$$\Psi = \sum_{[K], \nu}^{K_{max}, \nu_{max}} c_\nu^{[K]} e^{-\rho/2} \rho^{n/2} L_\nu^n(\frac{\rho}{b}) [\mathcal{Y}_{[K]}^\mu(\Omega) \chi_{ST}^{\bar{\mu}}]_{JT}^a$$

$$\Psi = \sum_{[K], \nu}^{K_{max}, \nu_{max}} c_\nu^{[K]} \underbrace{\psi_{[K], \nu}}_i$$

Asymptotic $e^{-a\rho}$ $\rho \rightarrow \infty$

$$H|\Psi\rangle = E|\Psi\rangle$$

$$\langle \psi_j | \times H \sum_i^N c_i |\psi_i\rangle = E \sum_i^N c_i |\psi_i\rangle$$

$$\sum_i^N \underbrace{\langle \psi_j | H |\psi_i\rangle}_{H_{ji}} c_i = E \sum_i^N c_i \underbrace{\langle \psi_j | \psi_i\rangle}_{\delta_{ji}}$$

$$\boxed{H\mathbf{c} = E\mathbf{c}}$$

Eigenvalue problem:
Diagonalization of big matrix