Nuclear Reactions and Astrophysics: a (Mostly) Qualitative Introduction

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Key Concepts Lecture 2013
Introduction

To observe the nucleus, we must use radiation with a (de Broglie) wavelength shorter than the features we wish to study.

How large is the nucleus?

\[ \lambda = \frac{h}{p} \]

Can we use electromagnetic radiation?

Nuclear reactions are crucial for studying nuclear structure.
Notation

Lexicon: \( T = \text{target}, \ b = \text{projectile}, \ R = \text{recoil}, \ e = \text{ejectile} \)

\[ T + b \rightarrow R + e, \text{ or equivalently } T(b,e)R \]

Specifically, \(^A\)Chemical Symbol, where \( A = Z + N \) is the atomic mass number

Some light nuclei have their own abbreviations: \(^1\)H = p, \(^2\)H = d, \(^3\)H = t, \(^4\)He = \( \alpha \); photons are \( \gamma \); excited nuclei denoted with an asterisk, e.g, \(^{12}\)C*
Types of Reactions

- **Elastic Scattering**: internal states and identities of nuclei unchanged
  
  E.g., the 1909 Geiger-Marsden experiment
  \[ ^{197}\text{Au} + \alpha \rightarrow ^{197}\text{Au} + \alpha \]

- **Inelastic Scattering**: One or both nuclei are excited, typically only the recoil
  
  E.g., \( ^{208}\text{Pb} + p \rightarrow ^{208}\text{Pb}^* + p \)
Reaction Classes

- **Transfer Reactions**: Both the projectile and target are transmuted by a transfer of one or more nucleons.
  - E.g., Rutherford, 1919: $^{14}\text{N}(\alpha,p)^{17}\text{O}$
  - E.g., pickup: $^{20}\text{Ne}(^{3}\text{He},^{4}\text{He})^{19}\text{Ne}^*$
  - E.g., stripping: $^{20}\text{Ne}(d,p)^{21}\text{Ne}$

- **Radiative Capture Reactions**: The projectile and target fuse with the emission of a photon.
  - E.g., $^{12}\text{C}(n,\gamma)^{13}\text{C}$
More Reaction Types

- **Fusion Evaporation Reactions**: the projectile and target fuse, and the excited compound nucleus de-excites by evaporating nucleons and/or α’s
  
  E.g., \( ^{12}\text{C} + ^{12}\text{C} \rightarrow ^{24}\text{Mg}^* \rightarrow ^{20}\text{Ne}^* + \alpha \)

- **Spallation Reactions**: when the projectile breaks the target up into a relatively large number of reaction products
  
  E.g., \( ^{181}\text{Ta} + p \rightarrow ^{11}\text{Li} + \alpha + p + ^{166}\text{Er} \)
Some Other Reactions

Fission: when heavy target nucleus breaks up into roughly equal mass fragments (can happen spontaneously)

E.g., $^{235}\text{U} + n \rightarrow ^{90}\text{Sr} + ^{144}\text{Xe} + 2n$

Other Reactions

E.g., weak reactions: $p + p \rightarrow d + e^+ + \nu_e$
Conservation Laws

Several quantities are conserved by all the interactions known to mediate nuclear reactions (strong, weak, EM):

- Baryon Number $A = Z + N$
- $Z$ and $N$ not separately conserved
- Electric Charge (note that electrons do not typically participate in nuclear reactions)
- Energy (the sum of rest energy and kinetic energy, i.e. total relativistic energy)
Conserved Quantities II

- Linear Momentum (vector)
- Angular Momentum (vector sum of relative orbital and intrinsic or spin)
- Orbital and intrinsic angular momentum not separately conserved
- The strong and electromagnetic interactions (but not the weak) also conserve parity, the product of the intrinsic parities and that associated with relative orbital angular momentum \((-1)^\ell\)
Q Values

The energy released in a nuclear reaction is called the Q value.

The source is nuclear binding energy, or equivalently, rest energy $E_0 = mc^2$

$$Q = \Delta mc^2 = (m_{\text{initial}} - m_{\text{final}})c^2$$

Since total relativistic energy is conserved, in the CM system we have $Q = T_{\text{final}} - T_{\text{initial}}$; rest energy is transformed into kinetic energy and vice-versa.
Energetics

- If $Q > 0$, a reaction is exothermic
- If $Q < 0$, a reaction is endothermic

Endothermic reactions are characterized by threshold energies below which the reaction is impossible; in the CM system, the kinetic energy $T_{\text{initial}}$ must equal or exceed $-Q$ to drive the reaction; there is no threshold for $Q > 0$

Q for elastic & inelastic scattering?
Reaction Probability

- Measured using the differential cross section $d\sigma/d\Omega(\theta, \phi)$

- Given by fraction of incident particles scattered into $d\Omega$ divided by number of targets/unit area

- Depends on $\theta$; independent of $\phi$ if unpolarized

Total cross section $\sigma$ obtained by integrating over solid angle

- Depends on energy!
Cross Sections

Classical interpretation in terms of colliding spheres; b = impact parameter, R radii; no collision unless \( b \leq R_1 + R_2 \)

Geometric cross section is area of disk \( \pi(R_1 + R_2)^2 \)

1. Depends on both target and projectile
2. Roughly comparable to observed cross sections
3. Units are barns; 1 b = 100 fm\(^2\)
Coulomb Scattering

- EM force causes scattering between charged particles even in absence of nuclear force

- Described by Rutherford in 1911 using classical mechanics; named for him, used to deduce structure of atom

- Rutherford formula valid in non-relativistic quantum mechanics too

\[ \frac{d\sigma}{d\Omega} = \left( \frac{Z_1 Z_2 e^2}{4T} \right)^2 \frac{1}{\sin^4 \frac{\theta}{2}} \]
Inverse Reactions

Time-invariance of interactions implies that time-reversed reaction rates related by detailed balance theorem

\[
\frac{\sigma(i \rightarrow f)}{(2J_b + 1)(2J_T + 1)p_i^2} = \frac{\sigma(f \rightarrow i)}{(2J_e + 1)(2J_R + 1)p_f^2}
\]
Reactions are selective: not all reactions populate the same energy levels or states.

Yield to different states reflects underlying structure as well as spin and parity selection rules.
Resonances

- When energy corresponds to state in compound nucleus, cross section is enhanced
- These enhancements are resonances
At low ejectile energies (high compound nuclear excitation energies) one sees evaporation products.

At high ejectile energy one sees discrete resonances corresponding to excited states.
Angular Distributions

- Direct reactions: energy shared with one or a few nucleons, fast timescale;
  Compound nuclear reactions: energy shared with whole nucleus, longer timescale

- Direct and compound nuclear reactions have different characteristic angular distributions

- The compound nucleus decays independently from its formation
Different angular distributions result from different values of transferred orbital angular momentum.

Using conservation laws, angular distribution reveals spin & parity of states.
Nuclear reactions power the stars and synthesize the chemical elements. We observe the elemental abundances through starlight and meteorites, and deduce the physical conditions required to produce them (Burbidge, Burbidge, Fowler and Hoyle 1957).

Fusion reactions of light nuclei are often exothermic.
Reactions at Astrophysical Energies

- Coulomb repulsion strongly inhibits charged particle-induced reactions.
- Neutron-induced reactions are hindered only by the centrifugal barrier.
Stars are hot balls of gas powered by internal nuclear energy sources.

The pressure at the centre must support the weight of the overlying layers: gravity tends to collapse a star under its own weight; as it shrinks, the pressure, temperature, and density all increase until the pressure balances gravity, and the star assumes a stable configuration.

For gas spheres at least 1/10 the mass of the Sun, the central temperature becomes hot enough to initiate thermonuclear fusion reactions.

Nuclear reactions in the hot, dense core are the power source of the Sun and all other stars.
The Sun

- Solar centre is about 150 times the density of water (~8 times the density of uranium)
- Central pressure is > 200 billion atm
- Central temperature is 16 MK
Solar Interior

Solar plasma is to good approximation an ideal gas

Described by Maxwell-Boltzmann distribution of thermal energies
Coulomb Penetrability

For non-resonant s-wave capture below the Coulomb barrier, charged particle induced reaction probability governed by the Gamow factor.

Coulomb barrier for p+p reaction is hundreds of keV
Gamow Peak

- Resulting asymmetric distribution known as the Gamow peak, centred about the most effective energy for thermonuclear reactions.
- Is only 6 keV for pp reaction and 20 keV for $^7\text{Be}(p,\gamma)^8\text{B}$ reaction.
- Implies important role for theory in extrapolation from energies accessible in laboratory.

![Graph showing relative probability vs energy in keV]
Tools of the Trade

**DRAGON**
Detector of Recoils And Gammas Of Nuclear reactions

Recoil Detectors

- Final Focus Box
- Magnetic Quads
- Electrostatic Dipole
- Mass Slit Box
- Quads

IC/PGAC Stop
MCP Start

Gamma Array
Magnetic Quads
Magnetic Dipole
Charge Slit Box

Gas Target
How Does the Sun Generate Energy?

- **Diagram:**
  - **pp I:** $p + p \rightarrow ^2\text{H} + e^+ + v_e$
  - **pp I:** $p + e^- + p \rightarrow ^2\text{H} + v_e$
  - **pp II:** $^2\text{H} + p \rightarrow ^3\text{He} + \gamma$
  - **pp II:** $^3\text{He} + ^4\text{He} \rightarrow ^7\text{Be} + 2\text{p}$
  - **pp III:** $^7\text{Be} + e^- \rightarrow ^7\text{Li} + v_e$
  - **pp III:** $^7\text{Li} + p \rightarrow ^8\text{Be}^* + e^+ + v_e$
  - **pp III:** $^8\text{B} \rightarrow ^7\text{Be}^* + e^- + v_e$

- **Equation:**
  - $4 \text{H} \rightarrow ^4\text{He} + 2\text{e}^+ + 2\text{v}_e$

- **Description:**
  - 4 hydrogen nuclei (protons) are fused into a single $^4\text{He}$ nucleus, with the emission of 2 positrons and 2 neutrinos.
  - This is very efficient, releasing about 1% of the binding energy, an enormous amount of energy.
  - The vast majority of stars generate energy similarly, by fusing hydrogen into helium.
Rates of pp Chain Reactions

The image shows a graph depicting the rates of various pp chain reactions as a function of temperature. The axis labels are:

- **Temperature (GK)**
- **\( N_A \langle \sigma v \rangle (\text{cm}^{3} \text{mol}^{-1} \text{s}^{-1}) \)**

The graph includes multiple reaction pathways, such as:
- \(^{7}\text{Li} + p\)
- \(^{3}\text{He} + ^{3}\text{He}\)
- \(^{7}\text{Be} + e^-\)
- \(^{7}\text{Be} + p\)
- \(^{3}\text{He} + ^{3}\text{He}\)
- \(p + p\)
- \(^{3}\text{He} + p\)

The graph illustrates the rate coefficients for these reactions across different temperature ranges. The specific data points and their relationships are depicted on the graph, showing how the rates change with temperature.
How Long Can This Go On?

- Sun started as 71% H by mass, 27% He
- Gradually, H in the core is fused into He
- Presently, ~ 4.6 billion years after it started shining, the Sun’s core H is about 1/2 gone
- Will continue fusing H into He in the core for another 5 Gyr
- Then what?
Red Giant Phase

- Inert He core contracts, outer layers expand, Sun becomes red giant
- H fuses into He in a shell surrounding the He core
- Size and mass of inert He core continue to expand
- The more massive the core becomes, the more pressure is required to support it and the overlying layers, so temperature rises
- Eventually, becomes hot enough for He to fuse into carbon via 3 alpha reaction ($^4\text{He}+^4\text{He} + ^4\text{He} \rightarrow ^{12}\text{C}$)
- Some carbon fused into oxygen
After Core Helium Exhaustion

- Inert carbon core (the ashes of helium burning)
- He-burning shell
- He layer (the ashes of hydrogen burning)
- H-burning shell
- Hydrogen-rich envelope
- Core never gets hot enough to burn carbon
Planetary Nebula Phase

- Star ejects outer layers, enriching interstellar medium with elements produced during the star’s life.

- The core, composed predominantly of carbon, oxygen, and helium, becomes a white dwarf, a stellar cinder that gradually cools, no longer able to generate energy by nuclear reactions.
How About More Massive Stars?

After core helium fusion comes carbon fusion, oxygen fusion, & silicon fusion.

Creates an onion-like structure, with an inert iron core surrounded by progressively cooler burning shells and a hydrogen-rich envelope.
## Major Stellar Fusion Processes

<table>
<thead>
<tr>
<th>Fuel</th>
<th>Major Products</th>
<th>Threshold Temperature (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen</td>
<td>Helium</td>
<td>4 Million</td>
</tr>
<tr>
<td>Helium</td>
<td>Carbon, Oxygen</td>
<td>100 Million</td>
</tr>
<tr>
<td>Carbon</td>
<td>Oxygen, Neon, Sodium, Magnesium</td>
<td>600 Million</td>
</tr>
<tr>
<td>Oxygen</td>
<td>Magnesium, Sulfur, Phosphorous, Silicon</td>
<td>1 Billion</td>
</tr>
<tr>
<td>Silicon</td>
<td>Cobalt, Iron, Nickel</td>
<td>3 Billion</td>
</tr>
</tbody>
</table>
Supernova Remnant
~1/2 of chemical elements w/ A > 70 produced in the rapid neutron capture (r) process: neutron captures on rapid timescale (~1 s) in a hot (1 billion K), dense environment (>10^{20} neutrons cm^{-3})

The other half are produced in the slow neutron capture process
Core-collapse supernovae favoured astrophysical site; explosion liberates synthesized elements, distributes throughout interstellar medium; Abundances of \( r \) process elements in old stars show consistent pattern for \( Z > 47 \), but variations in elements with \( Z \leq 47 \), implying at least 2 sites.
Solar System Abundances

- BBN & H Burning
- He Burning
- Si Burning & α-Rich Freezeout (Nuclear Quasistatistical Equilibrium)
- C Burning
- O Burning
- Neutron Capture
- Cosmic Ray Spallation