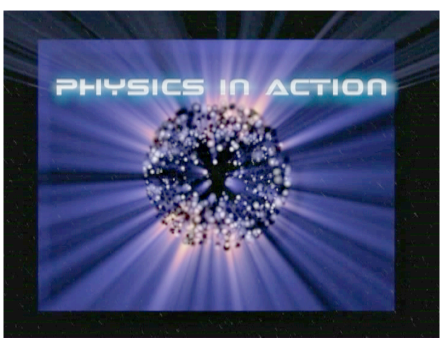
Demonstrating Special Relativity Using the TRIUMF Cyclotron



DVD Workbook

Teacher Edition

Contents

About Physics in Action

Funding

Funding provided by TRIUMF with a matching contribution from the Vancouver Foundation (http://www.vancouverfoundation.bc.ca/) and TRIUMF Technology Transfer:

Availability

The videos are available for free to any school in Canada that requests a copy. Copies of the videos may be requested online at http://www.triumf.ca

or by contacting the Outreach Coordinator at outreach@triumf.ca

Companion booklets are included on the DVD, or may be downloaded from the website. The electronic documents are freely editable by teachers as long as this page is included as is into the edited document.

TRIUMF is Canada's national laboratory for particle and nuclear physics.
Laboratoire national canadien pour la recherche en physique nucléaire et en physique des particules.
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Videos in the Physics in Action Series

Physics in Action is a new educational video series that will show high school students that the very same formulas and ideas they are studying in the classroom are in fact used everyday in a modern world-class subatomic physics facility they are not useless facts with no real-world value, but essential elements for scientific research! Students will be shown that the research conducted at TRIUMF, Canadas National Laboratory for Particle and Nuclear Physics, is not beyond their understanding, but actually lies well within their grasp. Physics in Action will become a valuable part of every high school physics teachers repertoire.

The videos are free to any school in Canada that wants one. They present a mix of live action, graphics, and 3D animation to help visualize the concepts, which often are difficult or impossible to convey in a typical classroom. Companion booklets will offer additional information and teaching resources for both students and teachers. The series has been developed in accordance with the prescribed learning outcomes of the BC and Alberta provincial education ministries.

In all, four educational videos were planned in the series.

Approaching the Speed of Light	Demonstrates the effects of Special Relativity on subatomic particle beams created at TRIUMF. Real data is provided, from which students can see clearly the speed-of-light limit, and how classical physics breaks down at high speeds. RELEASED 2004. Re-released in Winter 2010.
Electromagnetism and Circular Motion in a Cyclotron	Starts with hydrogen gas, and shows students how TRIUMF ionizes, steers, accelerates, and bombards it against a target to create exotic nuclei. Lesson modules demonstrate how each step can be understood using simple 11th and 12th grade electromagnetism. RELEASED Winter 2010.
Evolution of the Universe	Will take students on an exploration of the history of the universe from the Big Bang to the creation of our Solar System, showing where along the way the elements in their tin cup of water came from. IN PREPRODUCTION – Release Fall 2010.
Radioactivity	Will explain what radioactivity is and isn't, and demonstrate that it is a natural phenomenon with wide-ranging uses. Students will be shown the nuclear basis of alpha, beta, and gamma radiation. IN PLANNING – Release 2011.

Production Team

This Physics in Action video was produced by:

Stanley Yen, Ph.D.	TRIUMF Research Scientist
Marcello Pavan, Ph.D.	TRIUMF Outreach Coordinator
Phil Freeman	Richmond High School (Teacher Consultant)
Jerry Wong	Production, Animation
Brian Chan	Sound + Music
John Lambert	Gravity Lab Productions (Video Re-editing)
Ting Wang	Workbook Production and Graphics

Along with the valuable assistance of many members of the staff at TRIUMF and Richmond High School.

TRIUMF welcomes feedback from teachers and the public. If you have any questions, suggestions, or concerns about the Physics in Action series, please direct them to: TRIUMF Outreach Coordinator outreach@triumf.ca

Notes to the Teacher (from a teacher)

By using this video we hope you will be able to give your students a chance to see first hand the effects of relativity.

This video grows out of an actual field trip I was privileged to take my students on, which we found very memorable and helpful to them in learning about relativity. This year we used the video and though it was not as exciting as actually going to TRIUMF, it was still very interesting and helpful!

From my experience with this I would have the following general suggestions. Most of these will probably be obvious to you if you are an experienced teacher, but for those with less experience, or who like me can sometimes stand to have the obvious pointed out, Ill try to state all the little things that tripped me up the first time through! Additional suggestions you may have would be welcomed, and I hope can be included in future versions of this document, which will be maintained on the web at www.triumf.ca.

Pre-Teaching

The experiment could be presented as an introduction (to motivate teaching relativity) or after teaching students about relativity as a see it works! type of lab. I have used it mostly for the former, but in any case I feel it is important to stress that this is only one of a large set of experiments, all of which show the necessity for special relativity and all of which support its predictions. One of the hazards of teaching relativity is that, because of the lack of direct experience, students often see it as magical and some are resistant to the concepts, either because they find them too odd or because they dislike the consequences (in our heart of hearts a lot of us want faster than light spaceships too). One hope for this experiment is to show the reality of the divergence from classical theory in a real context. This needs to be followed up, of course, with a theoretical structure showing how the results flow from simple observed facts about lights and motion (or electromagnetic theory if you come at things that way).

Data

Students will need some guidance in finding the flight times for the particles. In particular they will be looking for three flight times (corresponding to the $\mathbf{e}^+$, $^+$, π^+ particles in the beam) but not all of these are necessarily visible on every plot. If only one peak is visible it is almost certainly the e^+, while if there are two peaks then the longer time will be $^+$. Finally when they see all three peaks then they are $\mathbf{e}^+$, $^+$, and π^+ from fastest (shortest time) to slowest (longest time). The reason for not seeing all the peaks is, of course, the decay of the pions (and muons), as well as factors involving the beam line that are probably best glossed over.

I would suggest using only the muons at first, and graphing pion data as an optional addition to the lab (along with finding rest masses perhaps). Some possible data tables for students are included, one with just the basic momentum, time, and velocity columns and one with γ and $\mathbf{v}\gamma$ columns if you are doing the linearized graph (described below).

Graph

As a first activity I found that simply calculating velocity and then plotting **p vs. v** was the best introduction for my grade 11 students. More advanced students also seemed to benefit from this, though some were ready to go straight to more complex graphs.

When having the students produce graphs you may want to have them plot both the data and the predictions of the two theories or you may want to have them plot the data on sheets that already show the curves so that they can see which theory the data fits as they plot it. I found that having the curves already there helped both with understanding and time, but some care had to be taken to explain the two different lines again as the students started the plotting.

If your students are used to plotting things in terms of dependent and independent variables then it is probably worth pointing out to the students that, although we are controlling the momentum and calculating the velocity, we *think of* the momentum as depending on the velocity, which is why we graph the momentum as the dependent variable in our graphs.

Later, perhaps after teaching about the relativistic transformations, the students may plot a graph of **p vs.** γ**v** to linearize the graph. This line will have slope $\mathbf{m_0}$, and generally gives quite good agreement with values obtained in other ways. There is a blank graph provided which is calibrated for this (called **p** γ**v**) that you can copy if desired.

Mass

The role of mass was a tricky one for my students. Some were under the impression that the experiments showed that mass changes, others that the experiment shows that mass is constant. Of course the trick, as discussed in the additional material, is that it all depends on what we call mass! I think it is important to choose one or another definition, acknowledge the other, but then stick with one interpretation. Students need to be encouraged to see this not as a conflict but as an example of how we can interpret things differently while still agreeing on the facts and even on the theory.

More details on the **m vs.** $\mathbf{m_0}$ issue is in a separate section on mass notation.

Units

I found that the use of the traditional SI units was most transparent for the students, although some of the stronger ones appreciated the value of the alternate units more common to particle physics. Using these alternative units, however, tended to confuse the majority of students.

This is a trade-off. Students find the numbers awkward, especially having to use scientific notation throughout. I find that encouraging them and acknowledging the awkwardness of these units helps them to deal with this better! The other thing that some students will have trouble with are graphing a value like 1.3 x 10^{-19} kg m/s when the axes of the graph are labeled x10^{-20} kg m/s. Id suggest showing them how to interpret this by rewriting, even if they should already know it!

Ive included a brief summary page on the experiment after this, which might be of some use to you! You may also contact me if you wish. If we get a lot of contact then we may try to create some sort of discussion forum (could be fun). I hope that you and your students find this experiment as interesting and helpful as we have!

Philip Freeman
pfreeman@Richmond.sd38.bc.ca
Richmond High School
April, 2005

Mass Notation

In this video we present the classical momentum as

$$\mathbf{p} = \mathbf{m}\ \mathbf{v} \qquad \textbf{(1)}$$

and the relativistic momentum as

$$\mathbf{p} = \boldsymbol{\gamma}\ \mathbf{m}\ \mathbf{v} \qquad \textbf{(2)} \qquad \text{where} \quad \gamma = 1/\sqrt{1 - \mathrm{v}^2/c^2}$$

By comparing (1) and (2), it is obvious that the change from classical physics to special relativity is accomplished by adding the factor of $\boldsymbol{\gamma}$ in front of **m**. Since $\boldsymbol{\gamma}$ is close to 1 for velocities much less than the speed of light, **c**, it is apparent that the classical formula (1) is just the small-velocity limit of the more general equation (2).

However, some textbooks present the relativistic momentum as

$$\mathbf{p} = \boldsymbol{\gamma}\ \mathbf{m_0 v} \qquad \textbf{(3)}$$

What is the difference? The answer is, there is NO difference! The mass $\mathbf{m_0}$ in equation (3) is exactly the same as the mass **m** in equations (1) and (2), and that is the rest mass of the object, the mass that the object has when it is sitting still.

The rest mass **m** (or $\mathbf{m_0}$, which is the same thing) for a proton is always 938 MeV/c^2, regardless of whether that proton is sitting still or moving at 90% the speed of light. To be make equation (3) consistent with equation (1) in notation, one should really write the classical momentum as $\mathbf{p} = \mathbf{m_0 v}$, but since nobody does that, we prefer equation (2) over equation (3) as the form for the relativistic momentum.

Now, in looking at equation (3), one could say that the momentum is behaving as if the mass **m** were increasing. That is, if we define the relativistic mass by $\mathbf{m_{rel}} = \boldsymbol{\gamma}\mathbf{m}$, then we could write the relativistic momentum as

$$\mathbf{p} = \mathbf{m_{rel}}\ \mathbf{v}$$

So the particle is effectively behaving as if the mass in the classical formula $\mathbf{p} = \mathbf{mv}$ were increasing. Thus, a particle can never reach the speed of light because as its speed approaches **c**, its relativistic mass approaches infinity and it becomes infinitely hard to accelerate it any further. In both cases we are recognizing that the old formula for momentum stops working.

When we write $\mathbf{m_0}$ we are blaming this failure on the mass, and saying that our old idea of mass has to be changed by adopting relativistic mass. There are some other reasons to do this too.

When we write just **m** we are saying that we dont really need a new idea for what mass is, but that we fix the momentum formula as a whole. There are some other reasons to do this too, and in fact this is the approach taken by physicists.

So whether we write **m** or $\mathbf{m_0}$ is mostly a matter of taste. We chose to use **m** as being most similar to the use of mass that you are probably most familiar with.

Units

Mass

In this video we expressed the mass of the particles in units of MeV/c^2. You are used to seeing mass expressed in units of kilograms (kg) so where does this new unit come from? One reason particle physicists use these units is that the mass of these particles is extremely tiny. If you look up the mass of the electron in your textbook youll see its mass in kilograms is 9.11×10^{-31} kg. Rather than use this absurdly small number physicists use the mass of 0.511 MeV/c^2 (your textbook may also list the mass this way), where 1 $MeV/c^2 = 1.79 \times 10^{-30}$ kg.

Energy

Einsteins famous equation $\mathbf{E = mc^2}$ relates a particles mass to its energy. We can see that using the mass of an electron in kilograms wed get its energy as:

$E = mc^2 = (9.11\times 10^{-31}\text{kg})(3.0 \times 10^8 \text{ m/s})^2 = 8.2 \times 10^{-14}$ J

This is again another extremely tiny number. If however we use the mass of the electron as 0.511 MeV/c^2 we can get its energy as:

$E = (0.511 \text{ MeV/c}^2)(c^2) = 0.511$ MeV where 1 MeV $= 1.6 \times 10^{-13}$ J

Now you see that we have a new unit for energy. You are used to expressing energy in joules (J). Particle physicists express energy in units of electron volts (eV). An electron volt is the amount of energy that an electron gains when it passes through one volt of potential difference. You may not have studied potential difference yet so lets look at a simple example. The AA batteries that you probably use in your portable CD player or digital camera have a potential difference of 1.2 V. So each electron that passes through this battery gains 1.2 eV of energy. To get the electrons up to the 120 MeV that we saw in the video wed have to put 100 million of these batteries end to end!

Momentum

Another new unit was used to describe the particles momentum. Momentum is found using the formula $\mathbf{p = mv}$. If we had an electron traveling at 5% the speed of light we would calculate its momentum as:

$p = mv = (9.11 \times 10^{-31}\text{kg})(1.5 \times 10^7 \text{ m/s}) = 1.37 \times 10^{-23}$ kg m/s

So again we see yet another tiny value and the traditional units of kg m/s. If we instead use the mass of the electron as 0.511 MeV/c^2 we can see that it is much easier to calculate its momentum.

$p = mv = (0.511 \text{ MeV/c}^2)(0.5c) = 0.256$ MeV/c

Conversion Factors:

1 MeV/c^2	=	**1.79×10^{-30} kg**
1 eV	=	**1.6×10^{-19} J**
1 MeV/c	=	**5.36×10^{-22} kg m/s**

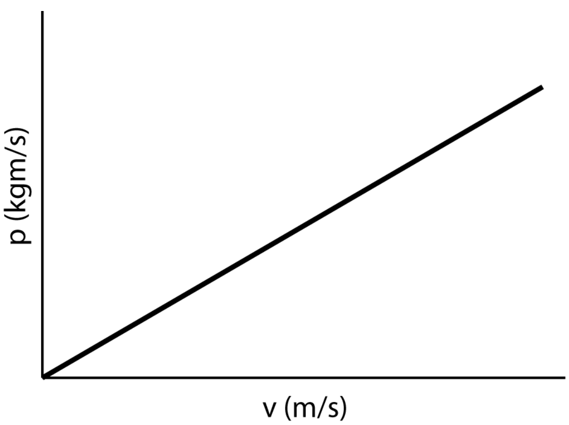
Approaching the Speed of Light

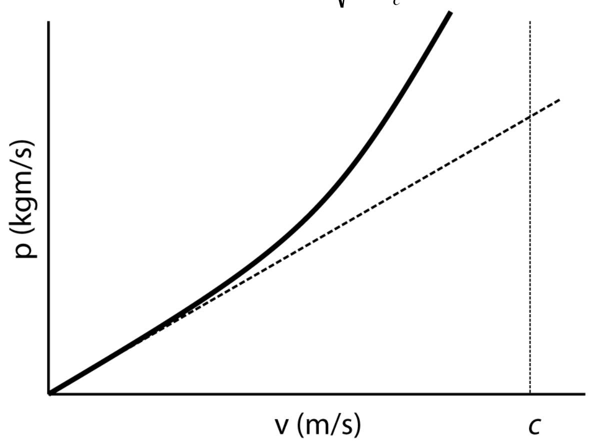
General Idea

We will be testing the predictions of classical mechanics and special relativity, in as far as they deal with momentum. If classical mechanics were completely correct we would expect to find that momentum was proportional to velocity:

$\vec{p} = m\vec{v}$

$\vec{p} = \gamma m\vec{v}$ where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$





If, on the other hand, relativity is correct, then increasing the momentum will not always correspond with an equal increase in velocity. The mass increases too, causing momentum to peel off from the straight line and asymptotically approach infinity as **v** approaches **c**, the speed of light.

Note that sometimes, it is written $\vec{p} = m_{rel}\vec{\mathrm{v}}$ using relativistic mass, though this approach is not favoured by physicists.

The Method

In our apparatus we can control the **momentum** of the particles by adjusting a magnetic field. We will also measure the time the particles take to travel a known distance, which gives us the velocity. Using this we can find what velocities correspond to what momenta. This will allow us to compare the predictions of classical and relativistic physics.

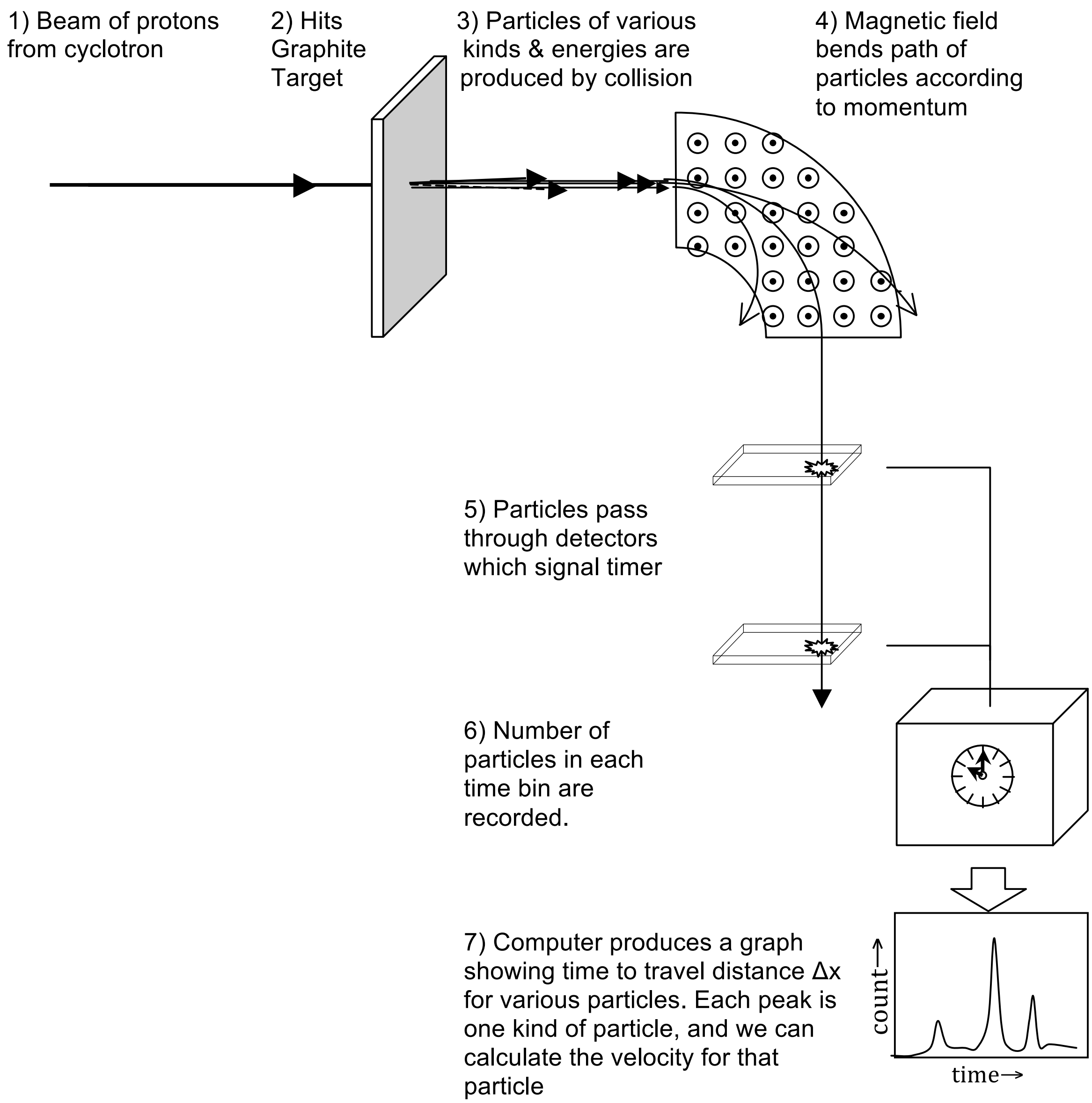
Momenta from 62 MeV/c to 220 MeV/c (or 3.31×10^{-20} to 11.8×10^{-20} kg m/s) can be selected. You will see simulations of how this is set up and how it runs, and you will be given real data sets from this apparatus. From this you may make your own graphs and see how the predictions of relativity stack up.

Units

It is common for scientists to choose units of measurement that result in numbers of common size, roughly 0.1 to 999. This makes them easier to remember and to do calculations with ones head (e.g. mass 0.5 MeV/c^2 and not 8.915×10^{-31} kg). Consequently, at TRIUMF the standard units used are not the ones that we are most used to in our high school (mechanics based) context. Although you will see the other units used in some parts of the video, you will be given values in the other familiar terms and you will find them rather awkward (all those powers of ten will be quite annoying)! Youll probably appreciate why particle physics use their own units, but we think it will make more sense to you if you use the units with which you and your students are accustomed (even if they are a bit awkward), to avoid confusion.

The Apparatus

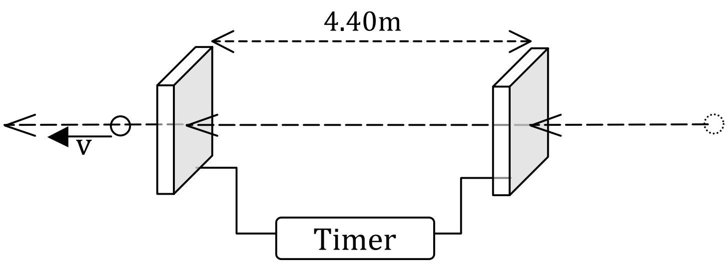
A schematic diagram showing the set up for the experiment is shown below.



Since we know the distance between the detectors and the time it takes the particles to travel between the detectors, we can find the velocity of each type of particle directly. We also know the momentum of the particles because only one momentum was bent correctly by the magnetic field. So we can see how momentum and velocity are actually related.

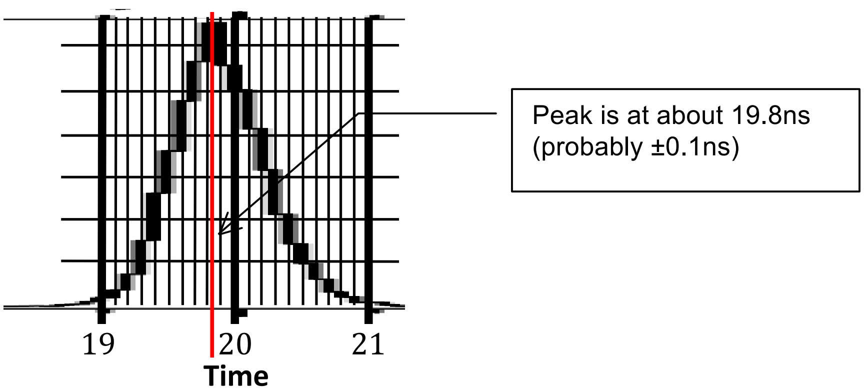
You will make a graph of observed vs. predicted values to compare the classical and relativistic theories with observation, and determine the rest mass of these particles.

Worksheet 1.1   Approaching the Speed of Light – MKS Units



Mass of muon: 106 MeV/c^2
= 1.89×10^{-28} kg
Momentum: 120MeV/c
= 6.41×10^{-20} kg m/s
Separation: 4.40 m

1) The picture below shows a simplified section of the data chart for 120 MeV/c, showing the muon peak. How much time (in nanoseconds) do the greatest number of muons take to cross the distance between the detectors? Notice that gridlines have been added. On the actual chart you can figure these out by looking at the steps!



(ans: 19.8 ns)

2) Given this time, what is the velocity of the muons?

$$v = \frac{\Delta d}{\Delta t} = \frac{4.40\text{m}}{19.8\times10^{9}\text{s}} = 2.22\times10^{8}\ \text{m/s}$$

(ans: 2.22×10^{8} m/s)

3) What momentum would you calculate given the muons velocity if the classical formula p = mv were correct?

$p = mv = (1.89\times10^{-28}\ \text{kg})(\ 2.22\times10^{8}\ \text{m/s}) = 4.20\times10^{10}\ \text{kg m/s}$ (ans: 4.20×10^{10} kg m/s)

4) What momentum would you expect given the muons velocity if the relativistic formula p = γmv were correct?

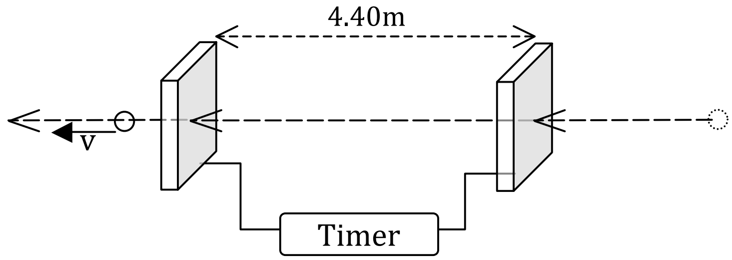
$$\gamma = \frac{1}{\sqrt{1-(2.22\times10^{8}\ \text{m/s} \div 2.9972458\ \times10^{8}\ \text{m/s}\ )^{2}}} = 1.490$$

so $p = \gamma mv = 1.490(1.89\times10^{-28}\ \text{kg})(\ 2.22\times10^{8}\ \text{m/s}) = 6.26\times10^{10}\ \text{kg m/s}$ (ans: 6.26×10^{10} kg m/s)

5) Which is closer matching the actual momentum, the classical formula or the relativistic formula?

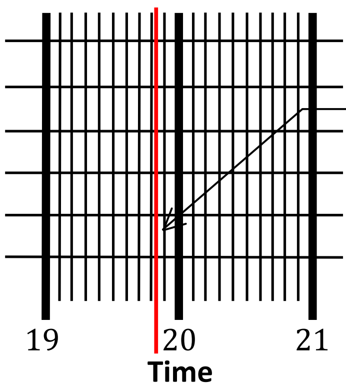
The stated value of the momentum (120 MeV/c) is 6.41×10^{10} kg m/s. Clearly the relativistic equation is a much better match than the classical formula.

Worksheet 1.1 Approaching the Speed of Light – Physicist Units

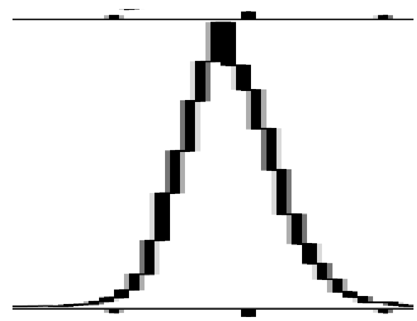


Mass of muon: 106 MeV/c^2
= 1.89×10^{-28} kg
Momentum: 120MeV/c
= 6.41×10^{-20} kgm/s
Separation: 4.40 m

1) The picture below shows a simplified section of the data chart for 120 MeV/c, showing the muon peak. How much time (in nanoseconds) do the greatest number of muons take to cross the distance between the detectors? Notice that gridlines have been added. On the actual chart you can figure these out by looking at the steps!



Peak is at about 19.8ns
(probably ±0.1ns)



(ans: 19.8 ns)

2) Given this time, what is the velocity of the muons?

$$v = \frac{\Delta d}{\Delta t} = \frac{4.40\text{m}}{19.8\times10^{9}\text{s}} = 2.22\times10^{8}\ \text{m/s}$$

since $c = 2.99792458\times10^{8}\text{m/s}$

$$v = \frac{2.22\times10^{8}\ \text{m/s}}{2.99792458\times10^{8}\text{m/s}} = 0.741c$$

(ans: 0.741c)

3) What momentum would you calculate given the muons velocity if the classical formula p = mv were correct?

$p = mv = (106\text{MeV/c}^2)(0.741c) = 78.6$ MeV/c

(ans: 78.6 MeV/c)

4) What momentum would you expect given the muons velocity if the relativistic formula p = γmv were correct?

$$\gamma = \frac{1}{\sqrt{1-(0.741c/c)^2}} = 1.490$$

so $p = \gamma mv = 1.490(106\ \text{MeV/c}^2)(0.741c) = 117$ MeV/c

(ans: 117 MeV/c)

5) Which is closer matching the actual momentum, the classical formula or the relativistic formula?

The stated value of the momentum is 120 MeV/c. Clearly the relativistic equation is a much better match than the classical formula.

Note that when comparing data where we have a number of points the best method is usually to graph the data not compare point by point. That way the whole pattern can be seen at once and differences from the predicted curve can be shown most clearly. Use the following sheets (or your own data table) to record the times and calculate the appropriate values for your plot.

Student Exercise Instructions

Using the data supplied on the bar graph for each momentum:

1. Determine **t** for the muon peak.
2. Use **d** = **4.40 m** and **v** = **d/t** to calculate **v**.
3. Plot **p** (vertical) versus **v** (horizontal).
4. Does the graph look curved (relativity) or straight (classical)?

Data Table [1]

Calculations

Momentum may be found on the graphs. Use the S.I. units.

Time is found from the peaks as shown in the video. Remember that these times are in nanosecond (10^{-9} s).

Velocity is $\frac{displacement}{time}$. The distance along the beam line is 4.40 m, so $v = \frac{4.40m}{time}$

Momentum (x10^{-20} kg m/s)	Time (x10^{-9} s)	Velocity (x10^{7} m/s)

Data Table [2]

Calculations

Momentum may be found on the graphs. Use the S.I. units.

Time is found from the peaks as shown in the video. Remember that these times are in nanosecond (10^{-9} s).

Velocity is $\frac{displacement}{time}$. The distance along the beam line is 4.40 m, so $v = \frac{4.40m}{time}$

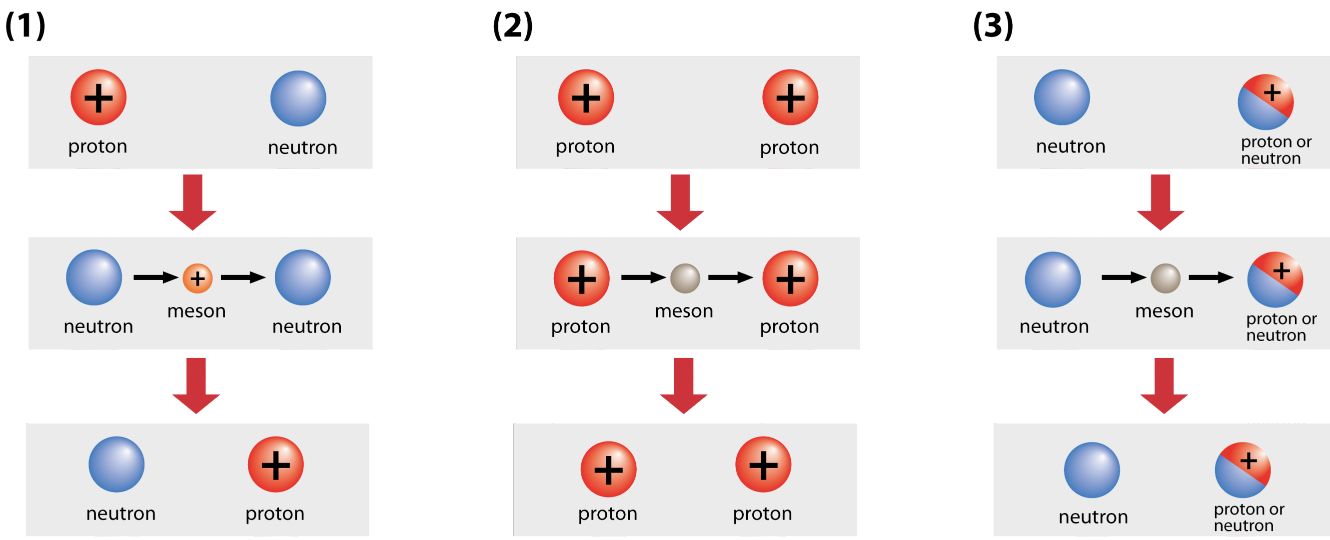
Momentum ($\times 10^{-20}$ kg m/s)	**Time ($\times 10^{-9}$ s)**	**Velocity ($\times 10^{7}$ m/s)**	$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$	v **x** γ

1.1 Extension: Introduction to Mesons and their Decays

The Strong Nuclear Force

It was known by 1932 that the atom consisted of negatively charged particles called **electrons**, orbiting around a positively charged **nucleus**. The nucleus in turn was known to consist of positive **protons** and electrically neutral **neutrons**. But what was holding the protons together? Their mutual electrostatic repulsion should make the protons repel each other. There *must* be some other type of force inside the nucleus.

The Japanese theoretical physicist Yukawa proposed that the nucleus was bound together by the sharing, or exchange of particles called **mesons**. This is illustrated in the following figure.



Visualizing the nuclear force as an “exchange” of a meson between protons and neutrons.

A proton could spontaneously turn into a neutron plus a positive meson. According to quantum mechanics, the heavier the meson, the shorter the time period that this is allowed to persist, because this intermediate state is a temporary violation of the conservation of energy. This meson is then absorbed by a neutron in the nucleus, and that neutron turns into a proton. The net result is that the initial proton has turned into a neutron, and the initial neutron has turned into a proton. A second possibility is that the proton turns into a proton plus a neutral meson, which is then absorbed by another proton. A third possibility is that a neutron turns into a neutron plus a neutral meson, when is then absorbed by another neutron or proton. In these last two possibilities, the initial protons and neutrons retain their identities.

The sharing of the meson pulls the proton and the neutron together, because the meson is a sticky particle that is attracted to both the proton and the neutron. This is somewhat like the situation of covalent bonds in chemistry, where two atoms are bonded together by sharing electrons, which are attracted to the positive nuclei of the two atoms being bound.

The picture of the atomic nucleus is thus not a static one, where there are just protons and neutrons sitting there unchanged, but a dynamic one, where mesons of various types are continually popping into existence and being exchanged between the protons and neutrons inside the nucleus.

The bonds that hold protons and neutrons together inside the atomic nucleus are roughly 1,000,000x stronger than the bonds that hold atoms together in molecules. That.s why nuclear reactions produce so much more energy than chemical reactions.

Meson Types and Their Production

Mesons were first discovered in collisions of high-energy particles from outer space called **cosmic rays**, with the atomic nuclei in the air molecules of the atmosphere. Later, with the invention of particle accelerators, mesons were discovered in man-made collisions of high-energy particle beams with nuclei of various materials. In these collisions, the high-energy particles would blow the atomic nuclei apart, and the mesons would be liberated as free particles. It was found that there was not just one type of meson, but a whole zoo of mesons. The accompanying table lists just a few of the dozens of known mesons. The lightest of these is the pi-meson (denoted by the Greek symbol $\boldsymbol{\pi}$), also called **pions** for short. It is found that pions come in positive, negative, and neutral types, and these are denoted by the symbols $\boldsymbol{\pi}^+$, $\boldsymbol{\pi}^-$, and $\boldsymbol{\pi}^0$, respectively. All these mesons live for a very short period of time before they decay into other particles. The table also shows the most important decay modes.

Symbol	Name	Mass (MeV/c^2)	Half-Life (seconds)	Decay Mode
π^+, π^-	charged pion	139.57	1.8 x 10^{-8}	$\mu + \nu_\mu$
π^0	neutral pion	134.98	5.8 x 10^{-17}	$\gamma + \gamma$
η	eta meson	547.30	3.9 x 10^{-19}	$\gamma + \gamma$ $3\pi^0$ $\pi^+\pi^-\pi^0$
ρ	rho meson	771.1	3.0 x 10^{-24}	$\pi + \pi$
ω	omega meson	782.57	7.1 x 10^{-23}	$\pi^+\pi^-\pi^0$

According to Einsteins famous equation $\mathbf{E = mc^2}$, in order to produce a particle of mass **m**, one needs to supply an amount of energy **E**. It is seen that the TRIUMF cyclotron, which produces a beam of protons of energy 500 MeV, has enough energy to produce only pi-mesons, and not the heavier mesons. In fact, TRIUMF is an acronym for Tri-University Meson Facility, a particle accelerator designed to produce large numbers of pi-mesons.

Meson Decay

The particle collisions produce large numbers of charged pions and neutral pions. The neutral pions decay so quickly that they dont even make it out of the atom, before they decay into two gamma rays:

$$\pi^0 \rightarrow \gamma + \gamma$$

Each of these high-energy gamma rays will strike other atoms in the carbon target, or the steel at the front end of the beam line, and produce an electron-positron pair.

$$\gamma + \text{atom} \rightarrow e^- + e^+ + \text{atom}$$

Here, $\mathbf{e}^-$ is the normal negatively-charged electron that we find in all atoms, and $\mathbf{e}^+$ is an anti-matter electron, which is positively-charged, and which is therefore named the **positron**.

The positively-charged pion decays into a positive muon (designated by the Greek symbol $\boldsymbol{\mu}$ [mu]), plus a muon-type neutrino (designated by symbol $\boldsymbol{\nu}$ [nu]).

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

whereas the negative charged pion decays into a negative muon plus a muon-type **anti**-neutrino:

$$\pi^- \rightarrow \mu^- + \overline{\nu_\mu}$$

The neutrinos and anti-neutrinos are ghostly particles, which are so inert that most of them pass straight through the concrete shielding blocks, and straight through the entire earth. They play no role in our experiment. Muons are particles that are similar to electrons, but about 200 times heavier than electrons. The muons also eventually decay into electrons (or positrons) plus neutrinos, but since their half-life is 1.5×10^{-6} seconds, and they are going near the speed of light, very few of them decay while flying down the beam line, which has a length of about 17 metres. (An ambitious student could estimate what fraction of the muons decay while flying down the beam line, with and without relativistic time dilation taken into account).

What enters the front end of the beam line, then, are the π^+ *and* π^- *which have not yet decayed, plus* μ^+ *and* μ^- *originating from decay of charged pions, plus* e^+ *and* e^- *made by the* γ *rays from decay of neutral pions.*

The bending magnets in the beam line select only positively charged particles, so what emerges from the end of the beam line to strike our detectors is a mixture of π^+, μ^+ and e^+, all with almost the same momentum determined by the strength of the bending magnets.

An examination of the data graphs shows that as the momentum of the beam line increases, the number of pions relative to muons also increases. This is because increasing the momentum of the beam line selects particles that are going faster, which in turn take less time to fly down the beam line, and hence the pions have less opportunity to decay into muons.

Standard Model

The discovery of mesons and other types of particles like the proton and neutron (as well as the muon and neutrinos) lead to such a zoo of particles that physicists felt these particles could not be fundamental after all, but must be made up of something else.

Murray Gel-Mann and others proposed a model based on the idea that protons, neutrons, pions and other particles were made up of a smaller fundamental particle called a quark. Quarks and Leptons (such as the electron and the muon) then become the fundamental particles, while protons or pions are composed of quarks according to particular laws of physics. For example the pions we have used are made up from an up quark and an anti-down quark (it sounds confusing, but it isnt that bad!)

This idea has been made into a sophisticated theory of subatomic particles now called The Standard Model which is very successful at helping us understand how particles behave and interact. Some tools of this theory, such as conceptual Feynman diagrams and energy calculations are often done in schools, and feature in the extension worksheet.
Materials about these topics can be found, for example, on the website of CERN (another particle physics research centre which TRIUMF sometimes works with). Try http://particleadventure.org to get started!

Worksheet 1.2 The Particles

The main particles you will see in this experiment are pions (π^+), muons (μ^+), and positrons (e^+).

These particles are produces when the high-energy protons in the beam hit the protons in the target:

$$p + p \rightarrow p + n + \pi^+$$

The π meson, or pion, has a mass of 139.6 MeV/c^2, or about 273 times that of an electron. It is a short-lived particle, with a half-life of only 17.7 ns in its rest frame.

Given that the mass of a proton is 938.3 MeV/c^2 and a neutron is 939.6 MeV/c^2.

1) What is the total rest energy of the two protons?

The total mass is 938.3 MeV/c^2 + 938.3 MeV/c^2 = 1876.6 MeV/c^2
so $E = mc^2$ = 1876.6 MeV/c^2 × c^2 = 1876.6 MeV (ans: 1876.6 MeV)

2) What is the total rest energy of the products of the reaction (the proton, neutrons, and pion)?

mass (p) + mass(n) + mass(π^+) = 938.3 MeV/c^2 + 939.6 MeV/c^2 + 139.6MeV/c^2 = 2017 MeV/c^2
so $E = mc^2$ = 2017.5 MeV/c^2 × c^2 = 2017.5 MeV (ans: 2017.5 MeV)

3) What additional energy must be supplied to create pions using the above reaction?

E_{final} $E_{initial}$ = 2017.5 MeV 1876.6 MeV = 140.9 MeV (ans: 140.9 MeV)

4) The energy needed for the reaction comes from the kinetic energy of the proton in the collision. Any energy not used in creating the particles remains as kinetic energy in the products. Given that the beam at TRIUMF can go up to 520MeV what is the highest kinetic energy that a pion produced this way could have?

$E_{Kf} = E_{Ki} - E_{needed}$ = 520 MeV 140.9 MeV = 379 MeV (ans: 379 MeV)

5) What is the total energy of the pion (including kinetic energy and rest mass)?

$E_{total} = mc^2 + E_K$ = 139.6 MeV/c^2 × c^2 + 379 MeV = 519 MeV (ans: 519 MeV)

6) What would a pions velocity be with this amount of kinetic energy? (hint: it is near c, use relativity!)

$$\gamma = \frac{Etotal}{E_o} = \frac{519\ \text{MeV}}{139.6\ \text{MeV}} = 3.715$$

$$v = \sqrt{1 \quad 1/\gamma^2} = \sqrt{1 \quad 1/3.715^2} = 0.963c$$

(ans: 0.963 c)

7) Why is the pion likely to have a range of kinetic energies, usually less than this maximum?

Some momentum and energy will be carried away by the other particles. With three particles there are a range of possible outcomes.

8) What would the momentum be for a pion with this maximum kinetic energy?

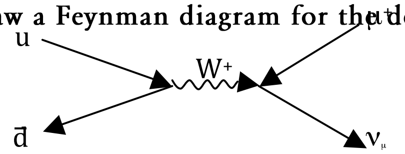
$p = \gamma mv$ = 3.715 (139.6 MeV/c^2)(0.963c) = 499 MeV/c (ans: 499 MeV/c)

Note: this second page of the worksheet was originally developed for the IB relativity and particle physics extension.

The pion decays into a muon and a neutrino: $\pi^+ \rightarrow \mu^+ + \nu_\mu$

The muon is a charged particle that is almost exactly like the electron, only about 200 times heavier (m = 105.7 MeV/c^2).

9) Draw a Feynman diagram for the decay of the pion (the π^+ consists of an up and an antidown quark).



10) If the muon received all the momentum of the original pion, what would its velocity be? (This is a bit trickier than it looks since you need to use relativistic formulas for momentum.)

$p = 499 \text{ MeV/c} = \gamma m v$ so $\gamma v = \dfrac{499 \text{ MeV/c}}{105.7 \text{ MeV/c}^2} = 4.725c$

thus $\dfrac{v}{\sqrt{1-(v/c)^2}} = 4.725c$ $v^2 = 22.3c^2(1\ \ v^2/c^2)$

$v^2 + 4.72v^2 = 4.72c^2$ $v = \sqrt{\dfrac{22.3c^2}{23.3}} = 0.978c$ (ans: 0.978c)

11) What would the total energy of a muon with that velocity?

γE_0 and $\gamma = \gamma v/v = 4.725/0.978 = 4.83$

$= 4.83(105.7\text{MeV}) = 510$ MeV (ans: 510 MeV)

12) Compare this to the kinetic energy of the pion originally. What is wrong?

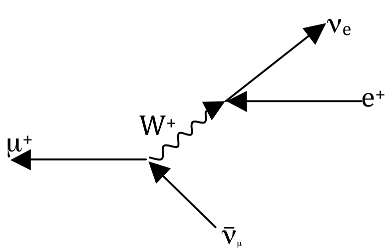
The original E_K of the pion was 519 MeV. About 9 MeV of energy is unaccounted for. Either energy is not conserved in this case, or something is missing!

13) Where is the missing energy?

Some other particle must take some energy and momentum after the decay. This is what the neutrino does, and the difference in energy in such decays is what tipped physicists off to the existence of the neutrino in the first place! The muon has a half-life in its rest frame of 1.5 microseconds, decaying into a positron plus two neutrinos: The electron is a very light particle, with a rest mass of 0.511 MeV/c^2 .

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$$

14) Draw a Feynman diagram for the decay of the muon (hint: the two neutrinos are a clue).



15) Given this very small mass the electron will have very high velocities. Explain why.

To carry a momentum even a fraction of that of the muon the electron must have a very very high velocity.

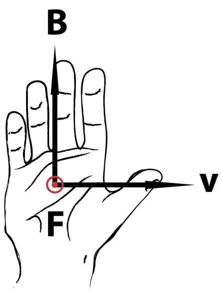
Extension: How does the Bending Magnet Select Particles for Momentum? [Right-Hand Rule Version 1]

A charge **q** moving in a magnetic field of strength **B** will experience a force of $\mathbf{F_B} = \mathbf{q\ v\ B}$

$\mathbf{F_B}$ = Force from the magnetic field
q = particle charge
v = particle velocity
B = magnetic field strength

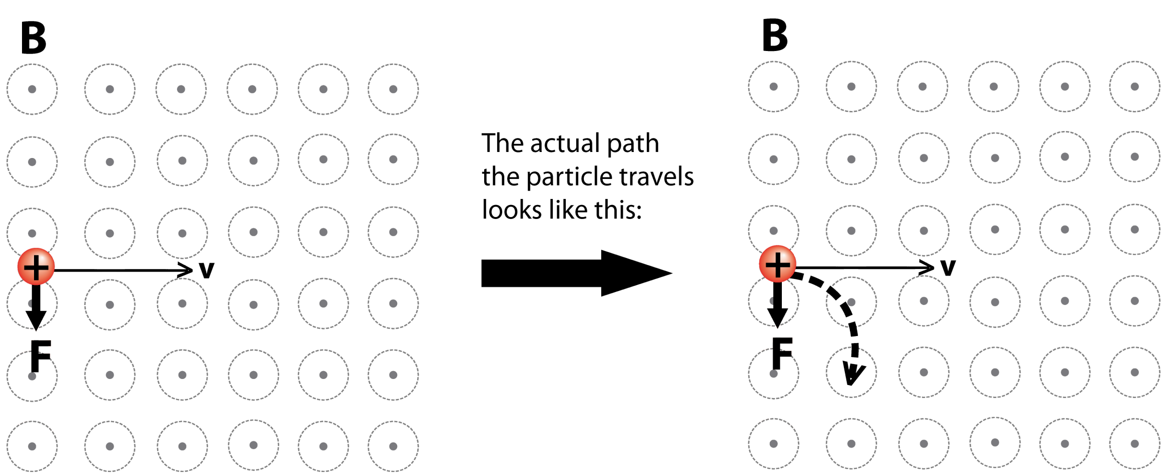
The direction of this force is given by the right hand rule:

Place your right hand on the table in front of you. If the particle is traveling in the direction of your thumb and the magnetic field is in the direction of the rest of your fingers then the force the particle experiences will be perpendicular to your palm.



(Force is coming out of the page)

For example lets suppose that there are magnets on either side of this piece of paper. This would lead to a magnetic field perpendicular to the paper and coming out of the page. A charged particle traveling into this field feels a force as shown below.



So a charged particle moving perpendicularly to a magnetic field will move in a circular motion. The acceleration for this circular motion is: $\mathbf{a = v^2/r}$

a = circular acceleration
v = particle velocity
r = radius of the circular path

and the force that is responsible for this motion will have magnitude: $\mathbf{F = ma}$

F = force due to acceleration
a = circular acceleration
m = particle mass

So the force that the magnetic field exerts on the particle must equal the acceleration force:

$$\mathbf{F = ma = mv^2/r = qvB}$$

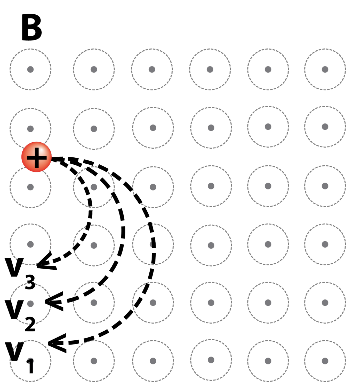
We can then solve for **r**:

$\mathbf{r = mv/qB}$ where **mv** is the non-relativistic momentum!

So, $\mathbf{r = p/qB}$

r = radius of the circular path
p = non-relativistic momentum
q = particle charge
B = magnetic field strength

So the radius of the circular motion of these particles depends on the particles momentum, the charge of the particle and the strength of the magnetic field. So particles with a larger momentum will travel in larger circles.



$\mathbf{v_1 > v_2 > v_3}$ Same mass, therefore different momenta!

Because the beam line admits only particles that bend at a certain radius, it selects for particles of a single momentum.

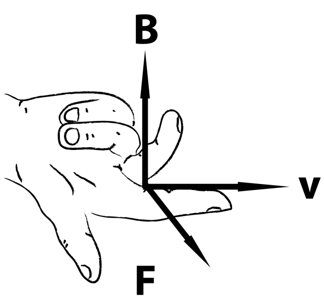
Extension: How does the Bending Magnet Select Particles for Momentum? [Right-Hand Rule Version 2]

A charge **q** moving in a magnetic field of strength **B** will experience a force of $F_B = q\ v\ B$

F_B = Force from the magnetic field
q = particle charge
v = particle velocity
B = magnetic field strength

The direction of this force is given by the right hand rule:

Take your right hand and make three right angels by making a L with thumb and pointing finger and then sticking your middle finger out at right angles. If the particle is travelling in the direction of your pointing finger and then magnetic field is in the direction of your middle finger, then the force is in the direction of your thumb.



(Force is coming out of the page)

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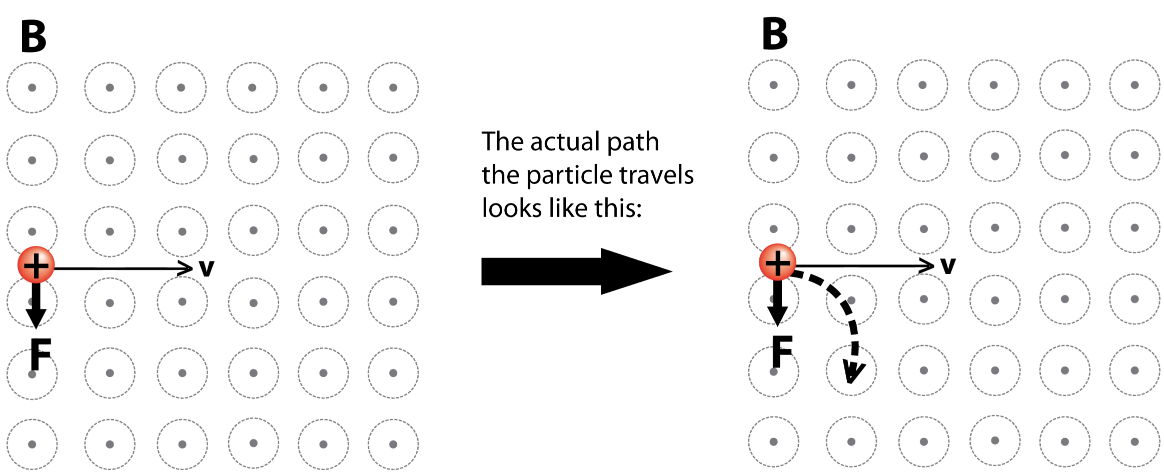

So a charged particle moving perpendicularly to a magnetic field will move in a circular motion. The acceleration for this circular motion is: $\mathbf{a = v^2/r}$

a = circular acceleration
v = particle velocity
r = radius of the circular path

and the force that is responsible for this motion will have magnitude: $\mathbf{F = ma}$

F = force due to acceleration
a = circular acceleration
m = particle mass

So the force that the magnetic field exerts on the particle must equal the acceleration force:

$$\mathbf{F = ma = mv^2/r = qvB}$$

We can then solve for **r**:

$\mathbf{r = mv/qB}$ where **mv** is the non-relativistic momentum!

So, $\mathbf{r = p/qB}$

r = radius of the circular path
p = non-relativistic momentum
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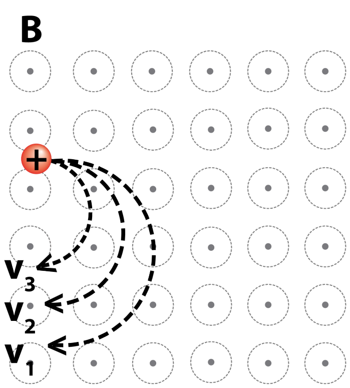
B = magnetic field strength

So the radius of the circular motion of these particles depends on the particles momentum, the charge of the particle and the strength of the magnetic field. So particles with a larger momentum will travel in larger circles.



$\mathbf{v_1 > v_2 > v_3}$ Same mass, therefore different momenta!

Because the beam line admits only particles that bend at a certain radius, it selects for particles of a single momentum.

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