Nuclear Forces and Mesons

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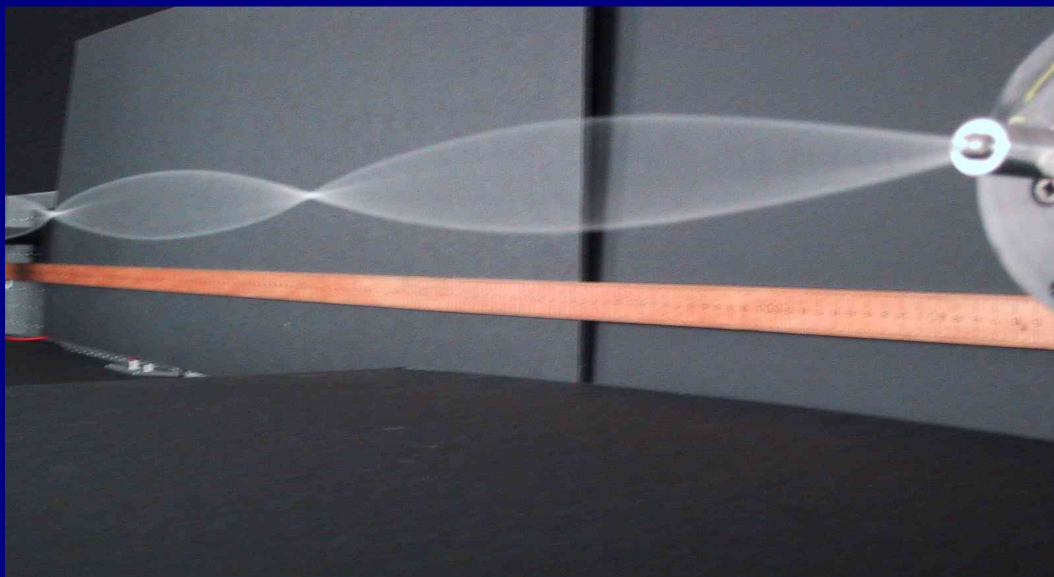
TRIUMF

On the scale of atoms, nuclei and subatomic particles, the quantum mechanical effects are large, and the behaviour of these systems must be obtained by solving the relevant equation describing the quantum waves.

These quantum waves are analogous to classical waves, e.g.

- the vibrations of a string (1-dimensional)
- the vibrations of the surface of a guitar or drum (2-dimensional)

Solving the relevant wave equations with the appropriate boundary conditions give the vibration modes of:



waves on a string

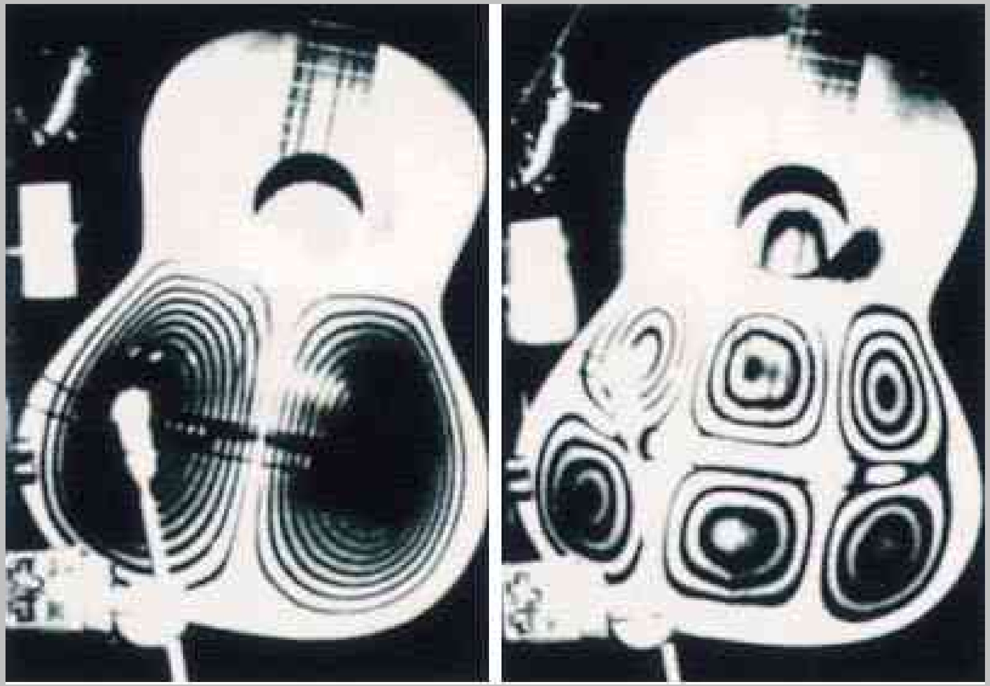
$$\boxed{\frac{\partial^2 u}{\partial t^2}(x,t) = c^2\,\frac{\partial^2 u}{\partial x^2}(x,t)}$$

$$c = \sqrt{\frac{T}{\rho}}$$

Here, u is the amplitude of the wave, and is a function of position and time

Similarly, for a 2-d surface, we solve the 2-d wave equation with the proper boundary conditions to get the permitted standing waves and frequencies, each of which displays nodes at specific locations.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$



Vibrating modes of
the surface of a guitar
at two different frequencies.

Chladni patterns formed by sand
on a vibrating metal plate.

Schrodinger:
Wave equation that describes quantum mechanical waves.
Fundamental to quantum chemistry and physics.

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r})+V(\mathbf{r})\psi(\mathbf{r}) = E\,\psi(\mathbf{r})$$

wavefunction ψ(x) describes the amplitude of the quantum-mechanical wave at position **r**

analogous to amplitude of the vibration on a string

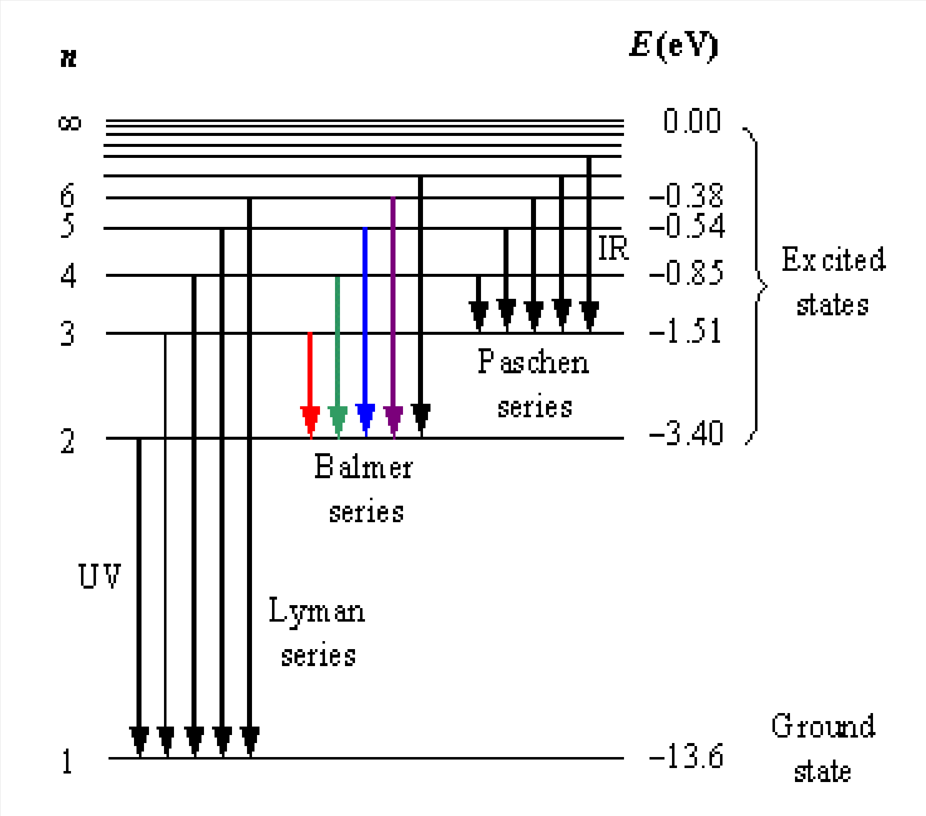
$|\psi|^2$ is the probability density for finding the particle at position **r**

depends on potential energy function V(r) --
a different potential energy V(r) results in a different wavefunction Ψ and different energy E

Hydrogen atom: electron sits in the Coulomb potential of the proton

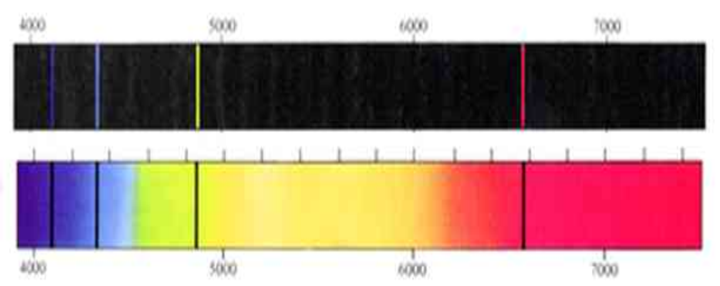
$V(r) = -k e^2 / r$

Use this in the Schrodinger eq. and solve for the wavefunctions ψ(x) and permitted frequencies f, just like for the strings and guitars.
Since Planck told us that E = h·f   there are only certain permitted energies E that the electron can occupy!



Energy levels of the hydrogen atom with some of the transitions between them that give rise to the spectral lines indicated.

Spectral lines from excited hydrogen atoms

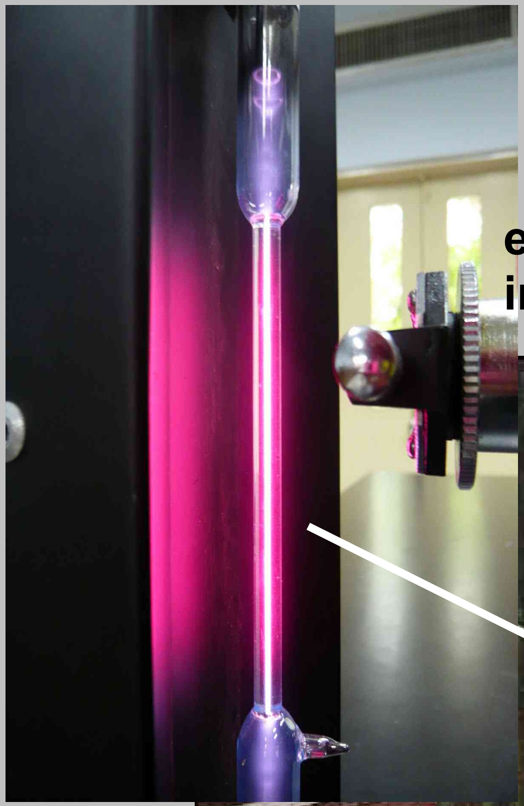


The Hydrogen Spectrum
Above: Emission
Below: Absorption

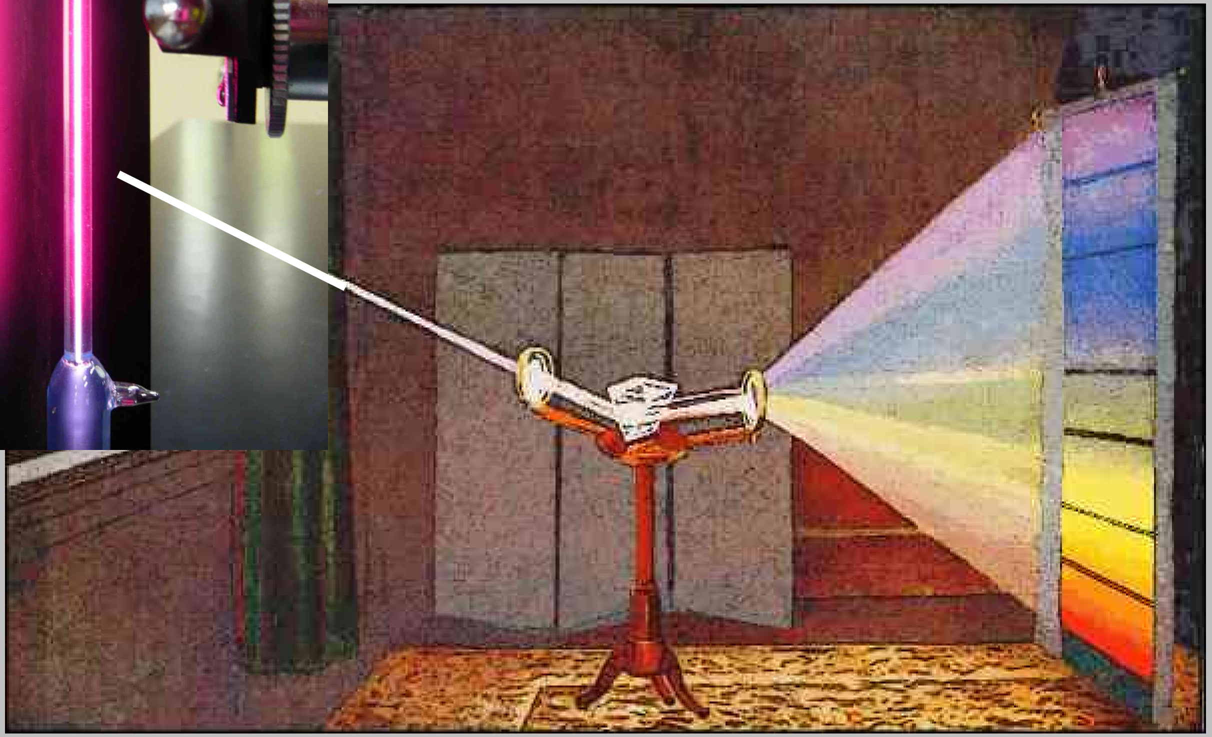
Positions of spectral lines would be different if potential V(r) were different. The spectrum of hydrogen tells us that the force holding the electron to the proton is the Coulomb force.

Experimental study of atomic spectra

electrical discharge in hydrogen lamp
excites the atoms

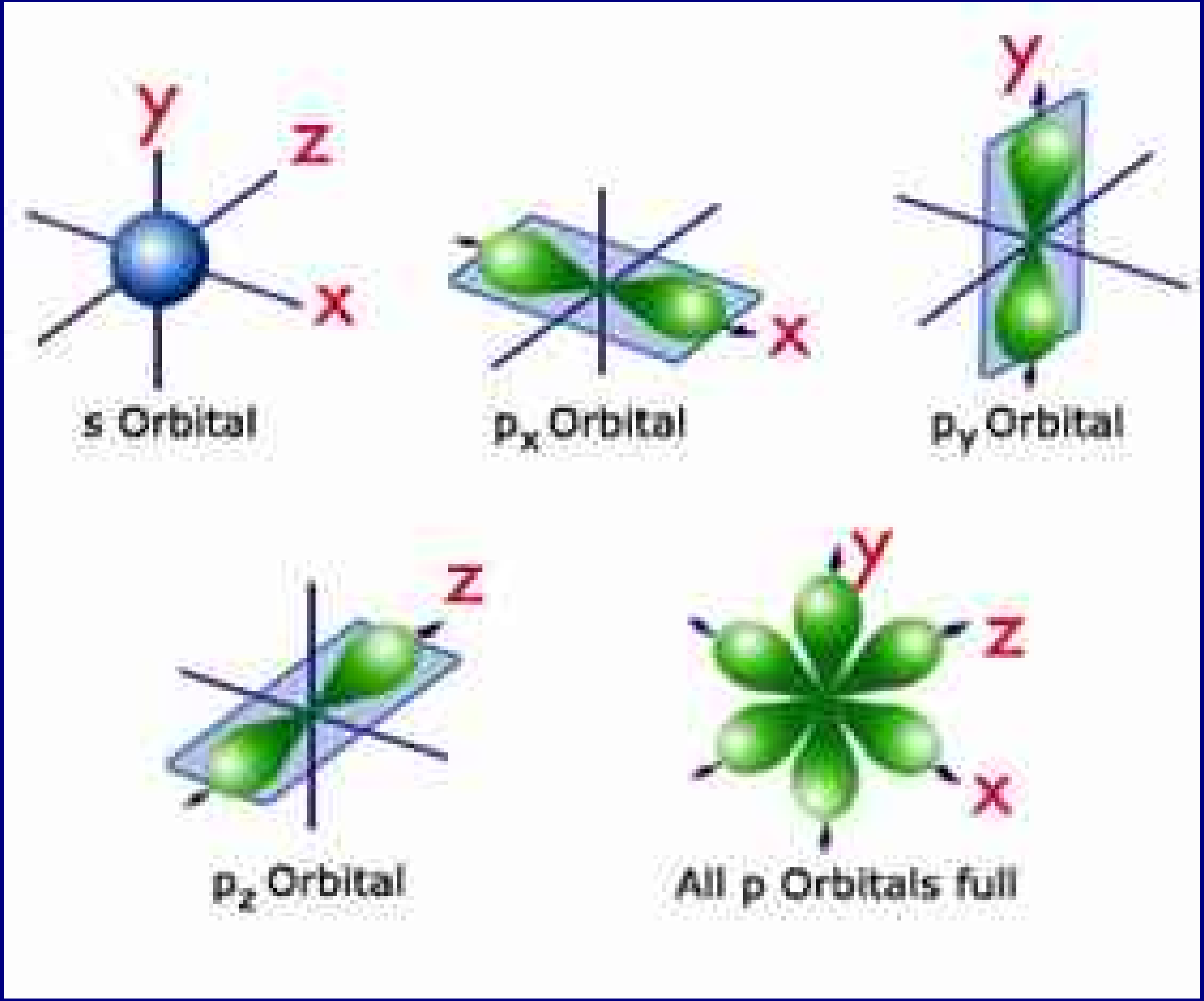


emitted light analyzed by a spectroscope
into its constituent wavelengths

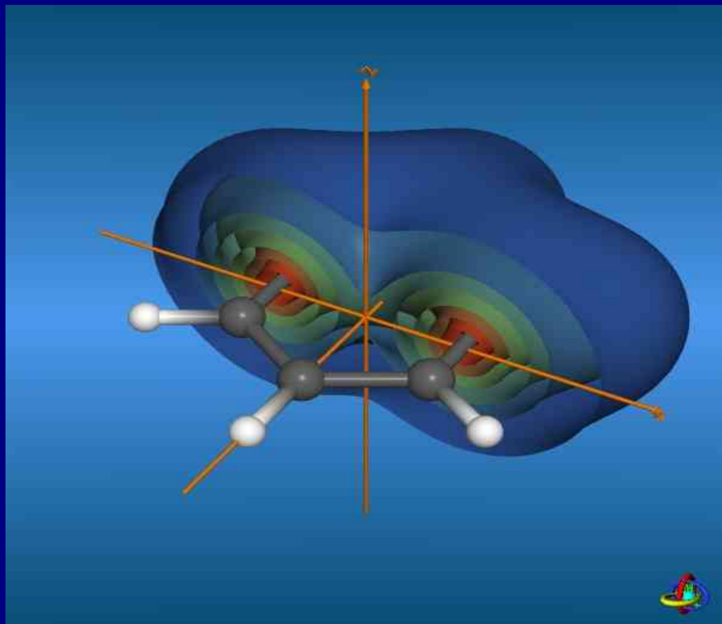


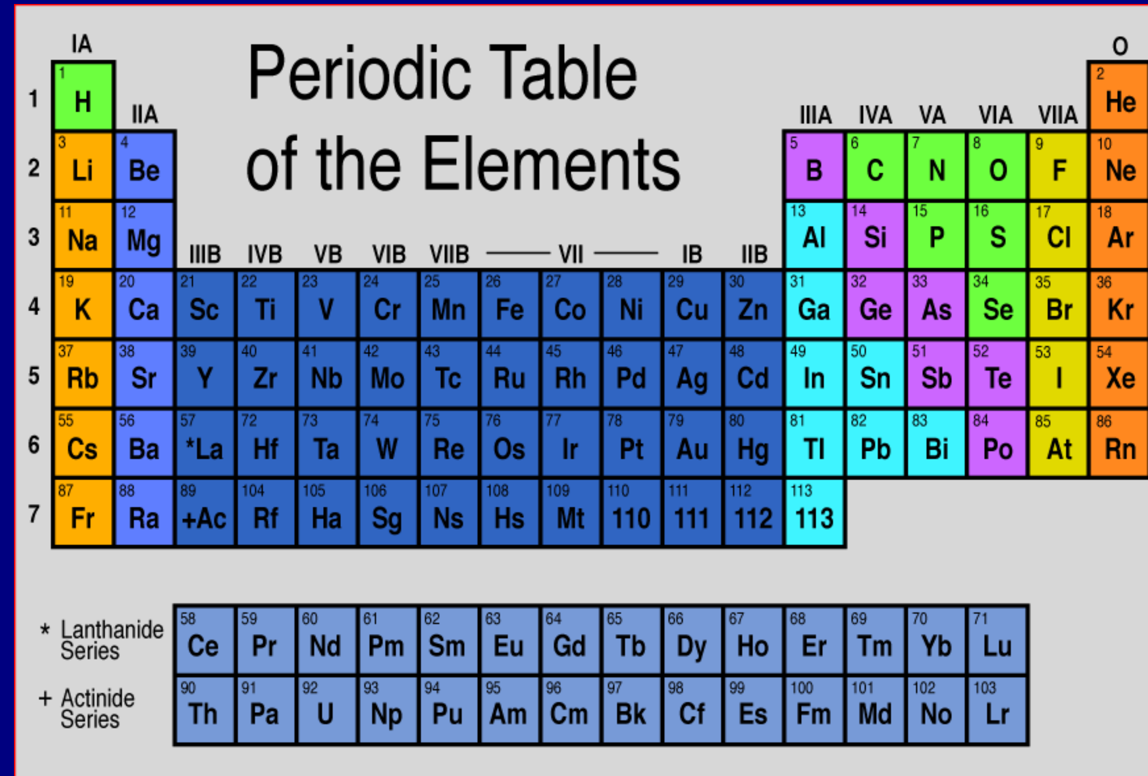
Something very similar will be done for studying nuclear energy levels!

$|\Psi|^2$ gives the probability density of the electrons in the atom (i.e. the electron orbitals)



This allows us to understand key features of chemistry:





molecular binding
shapes of molecules

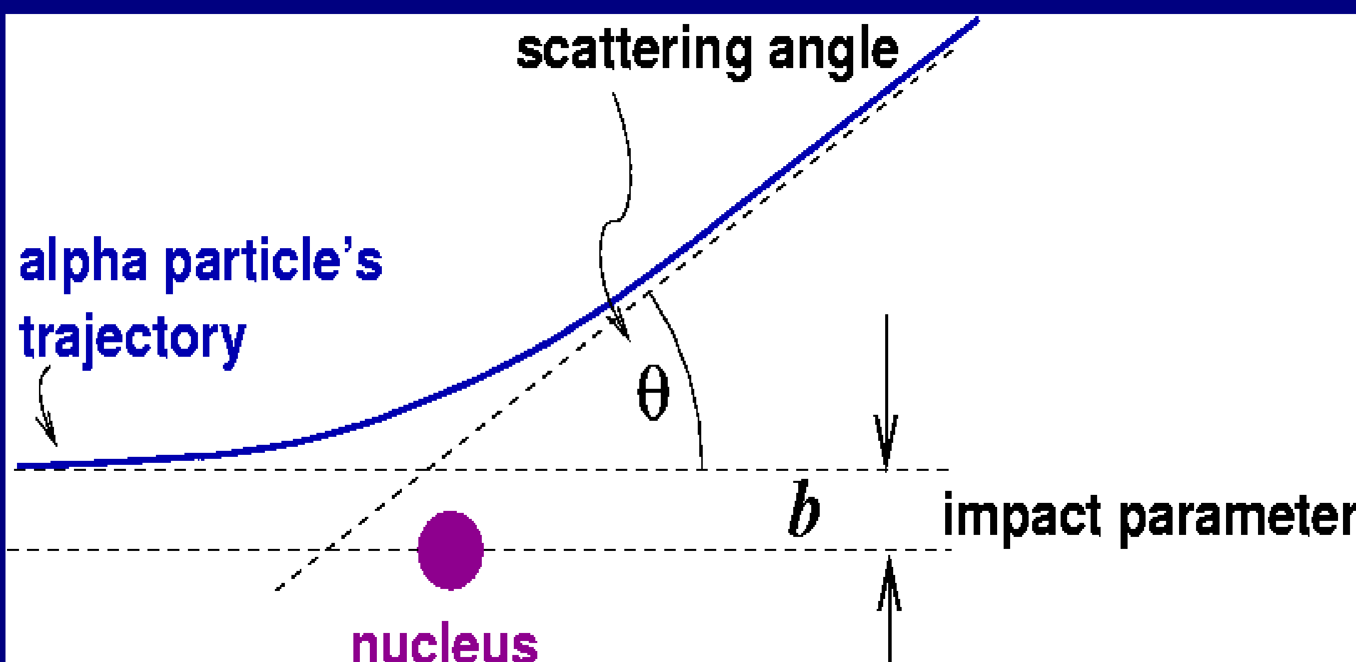
chemistry = Schrodinger's Eq. + Coulomb potential

At the scale of atoms and molecules (~ 10^{-10} m)
no evidence in atomic or molecular spectra of
anything except electromagnetic interactions!

The protons inside the nucleus <u>repel</u> each other electrostatically.
There must be some new, attractive force binding the nucleons together.
It must be a short-ranged force, because the nuclei in e.g. a water molecule do not feel anything except their mutual Coulomb replusion and the attraction of the electrons.

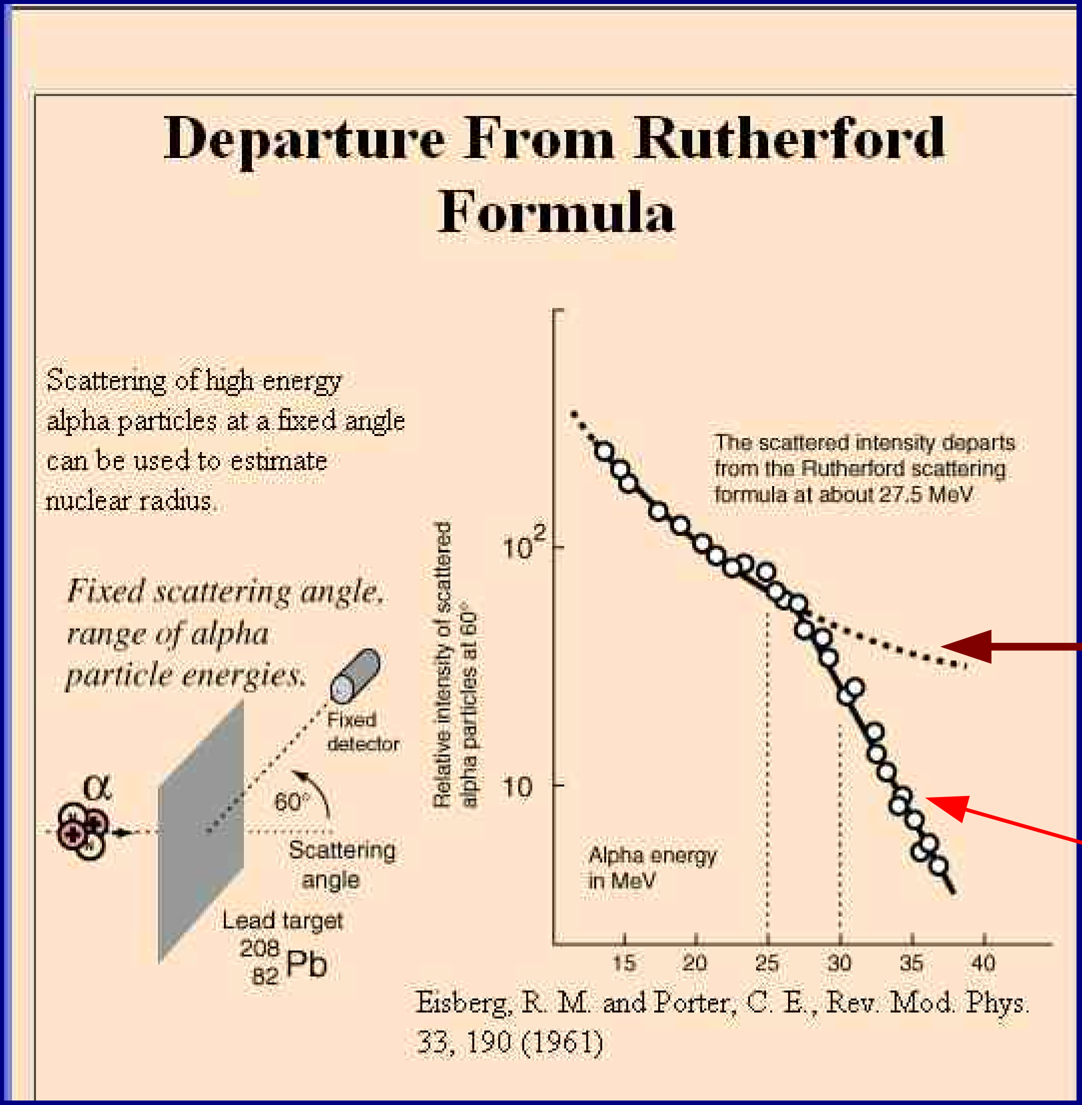
Consider Rutherford scattering. If only potential that the alpha particle feels is the Coulomb potential $V(r) = -kqQ / r$ then the probability of scattering through angle θ is given by

$$P(\theta) = (qQ)^2 / \{E^2 \sin^4(\theta/2)\}$$



Rutherford never observed any deviation with the low energy alpha particles he had available from radioactive sources, and concluded that the nucleus must be < 27 fm in size.

With higher energy alpha particles from an accelerator, we can see deviations from this formula.



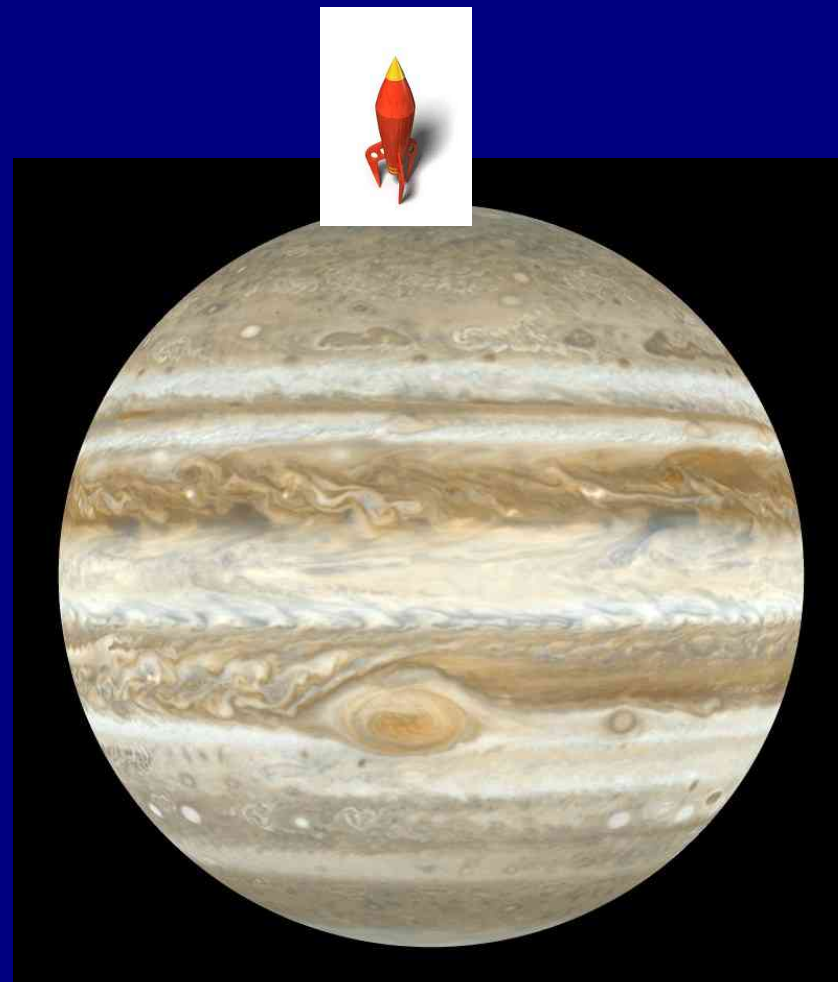
pure Coulomb potential

deviation at high energy (close approach) due to strong nuclear forces

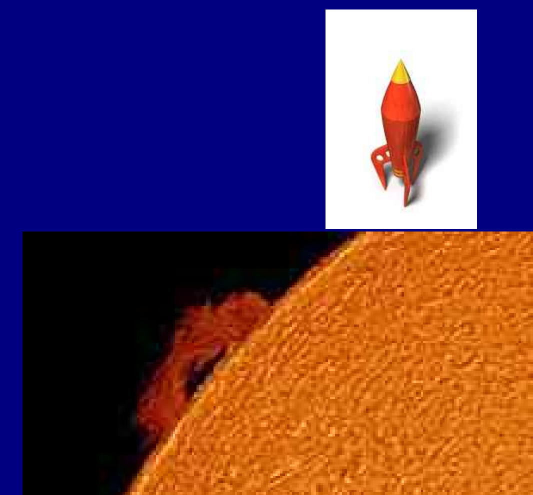
Consider the binding energy in the following two situations:

a) Binding energy of a rocket on the surface of a planet

Moon:
escape velocity
2.4 km/s



Jupiter:
escape velocity 59.5 km/s

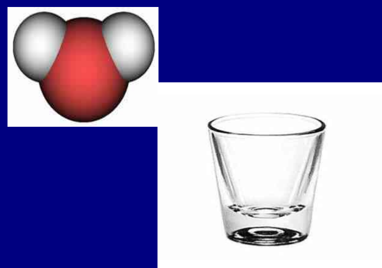


Sun:
escape velocity
617 km/s

$V_e = \sqrt{(2GM/r)}$ i.e. the larger the planet mass M, the more tightly the rocket is bound

The binding of the energy to the planet increases with the mass of the planet because gravity is a long-range force. The rocket feels the gravitational attraction of every part of the planet it is sitting on.

b) Vaporization energy of water – how much energy does it take to separate water molecules from a body of water?



from a shot glass of water: 540 kcal / mole

from a 2 litre kettle of water: 540 kcal / mole

from Lake Erie: 540 kcal / mole

The binding energy of water molecules to a mass of water does NOT depend on the mass of that water, because the binding forces are short-range Van der Waals forces.

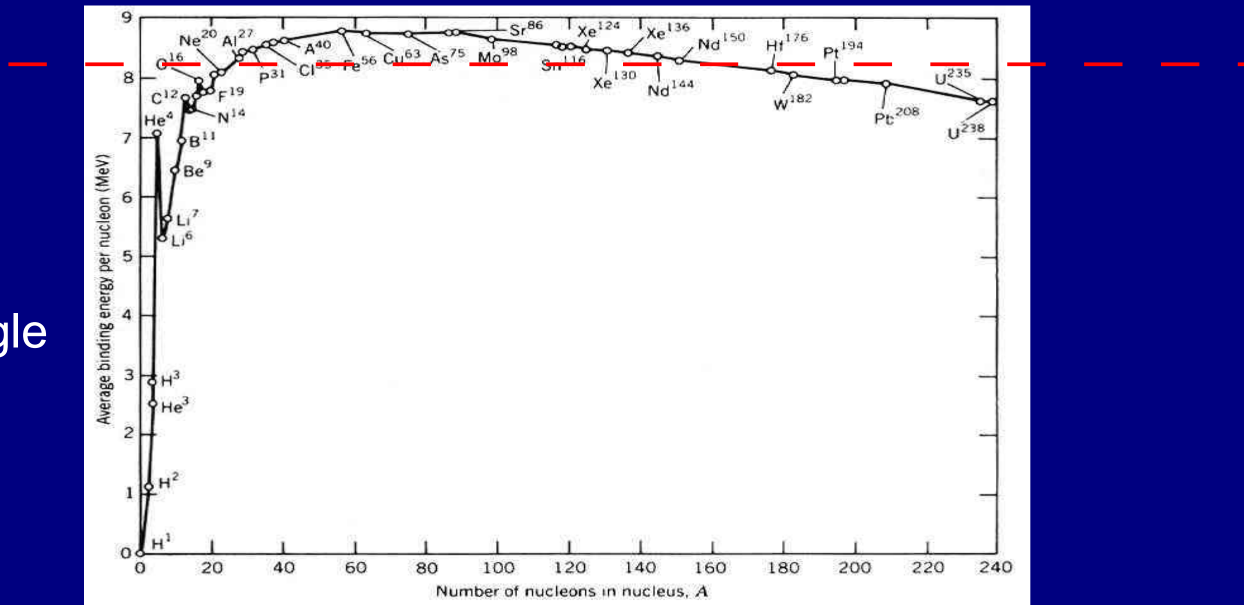
The water molecule feels the attraction ONLY of its nearest neighbours; it does not feel the attraction of far-away water molecules.

These two examples illustrate the difference in binding energy vs mass for long range forces (gravity) and short range forces (Van der Waals).

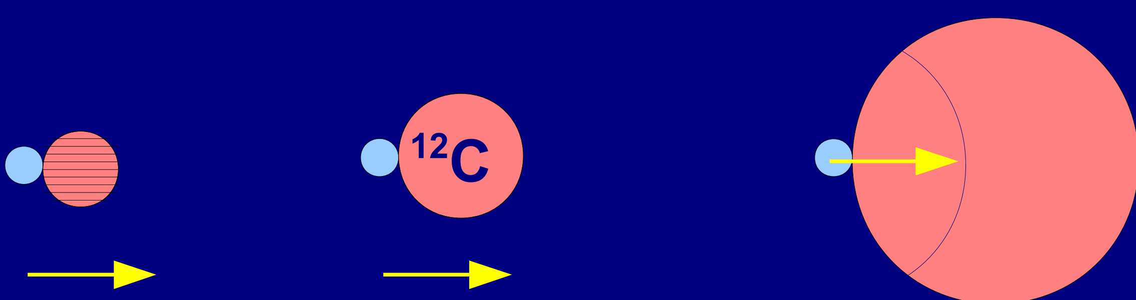
So, what about NUCLEAR forces?

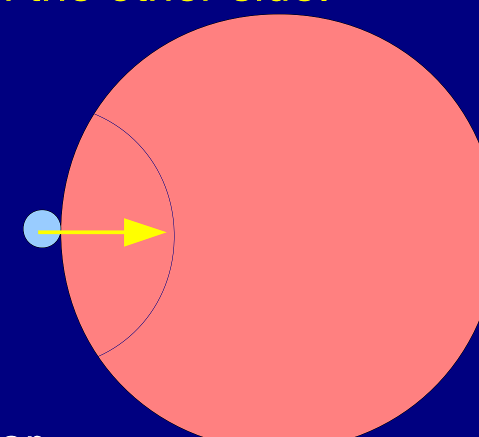
The binding energy of a nucleon first rises rapidly with atomic mass A, then starts to flatten out around mass 12 (“saturation of nuclear forces”).

Binding energy of a single nucleon versus mass



Each nucleon feels only its nearest neighbours, and for nuclei heavier than ^{12}C, the nucleons on one side of the nucleus don't feel the nucleons on the other side.





No extra binding if nucleus is larger than range of the nuclear force!

From electron scattering experiments, where we measure the width of the diffraction pattern for electrons scattering from the carbon nucleus, we know that the ^{12}C has a diameter of 5.4 fm, so the nuclear force must have a range of $\leq$ 5 fm

A more refined estimate give something closer to ~ 2-3 fm.

Since nuclei are composed of nucleons (protons and neutrons), we want information on the nucleon-nucleon (NN) potential. Two ways to study this: scattering experiments (free nucleons) or bound states (nuclei).

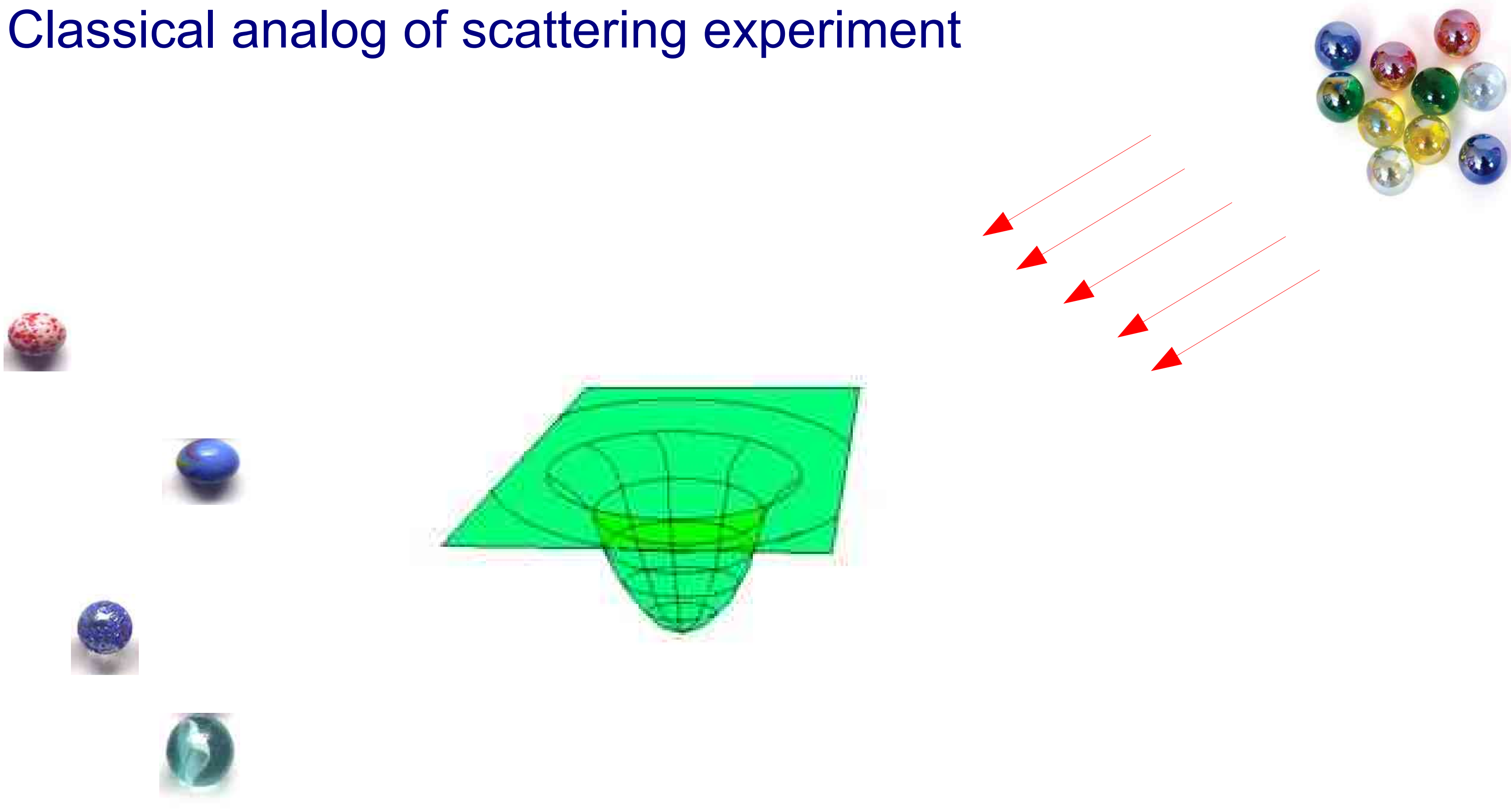
This is analogous to studying the Coulomb force between an electron and a nucleus: you can scatter electrons from a nucleus (free particles) → Rutherford scattering formula

or you can study the spectroscopy of the bound states of an electron and a nucleus (i.e. an atom)

Both will give you information about the shape and depth of the Coulomb potential between the electron and the nucleus.

Similarly, both scattering experiments and bound states of nucleons will tell us about the nuclear forces between protons and neutrons

Classical analog of scattering experiment



Angular distribution of the scattered marbles tells us the shape of the potential well that is scattering the projectiles away from their initial trajectories

Quantum mechanical:

Born Approximation Scattering Amplitude

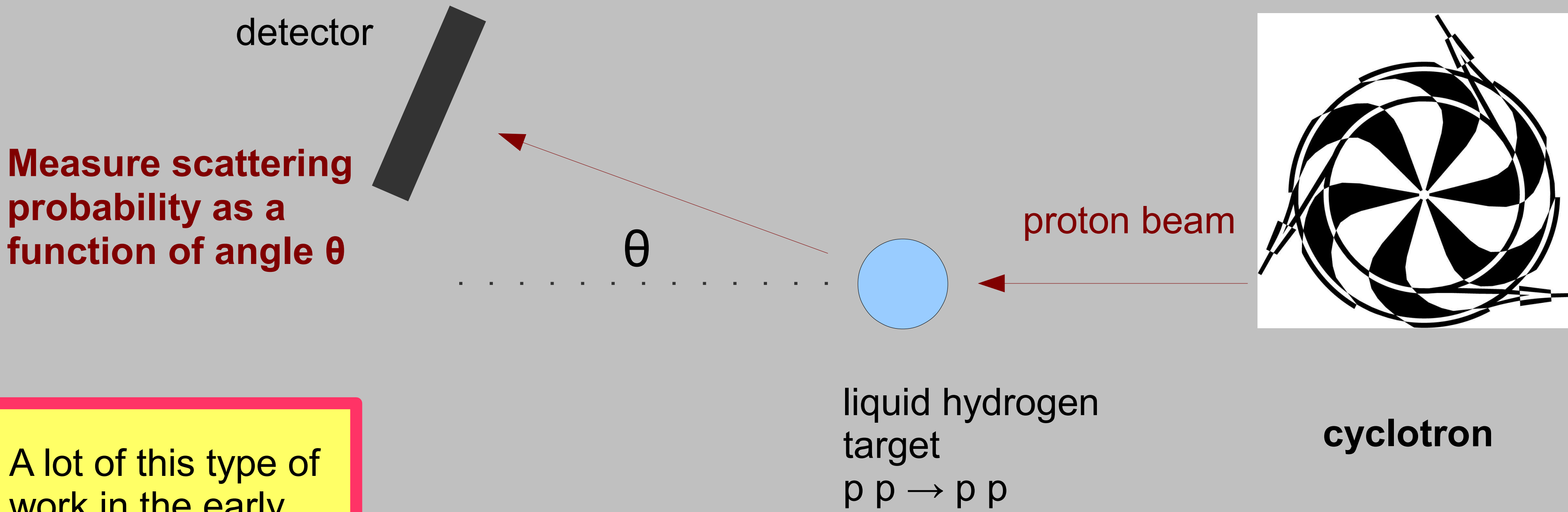
$$A = \int \psi^*_{final}\, V(r)\, \psi_{initial}\, dV = \int \exp(-i\mathbf{k_f}.\mathbf{r})\, V(r) \exp(i\mathbf{k_i}.\mathbf{r})\, dV$$

$$= \int \exp(i\mathbf{q}.\mathbf{r})\, V(r)\, dV$$ where $\mathbf{q} = \mathbf{k_i} - \mathbf{k_f}$ = momentum transfer in the scattering

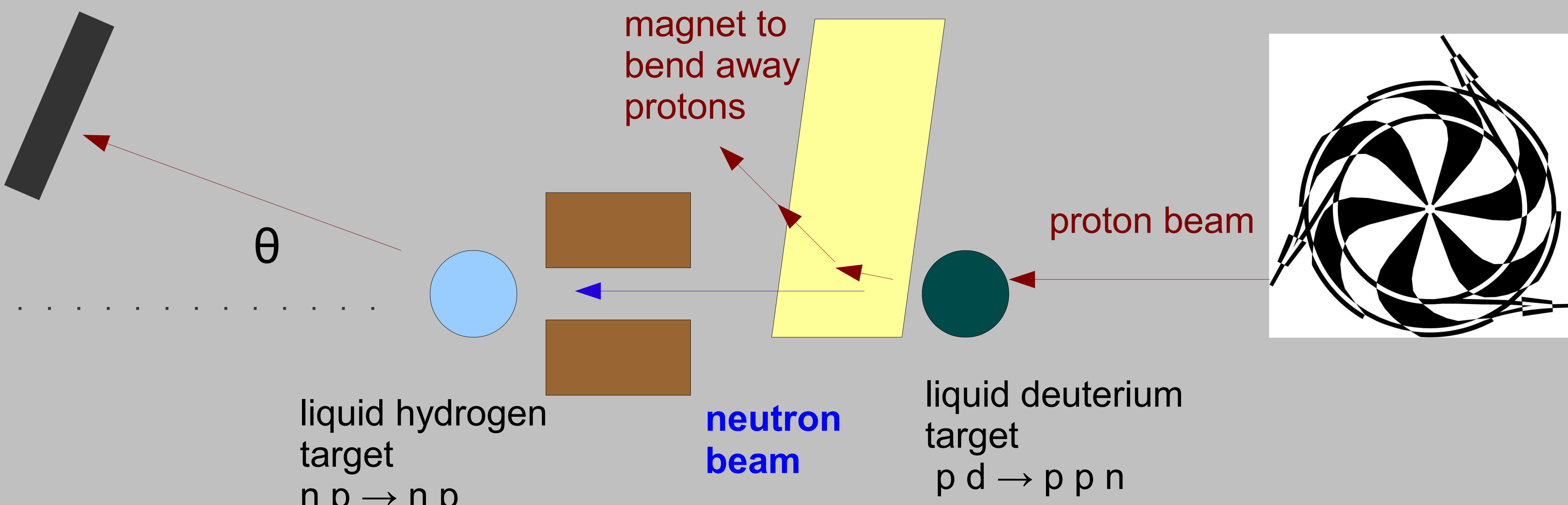
i.e. the scattering amplitude is the Fourier transform of the scattering potential V(r)

and the scattering probability is $|A|^2$ = the square of the Fourier tr.

Exactly what we saw last week when we looked at scattering pattern from targets of different sizes – the scattering pattern is the square of the Fourier transform of the charge distribution in the target.

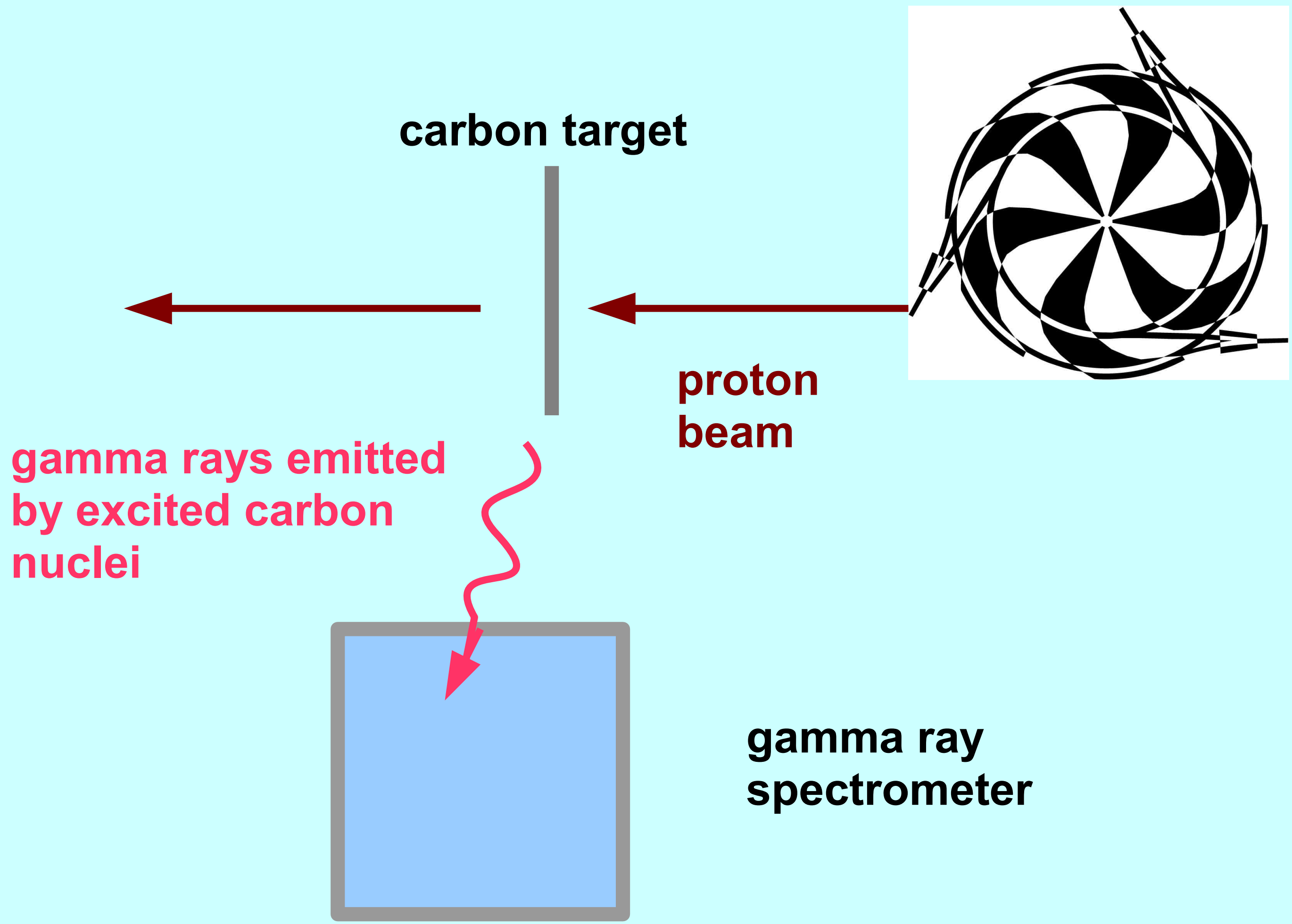


A lot of this type of work in the early days of TRIUMF!



Spectroscopy of Bound States (Nuclei)

Just like atoms and molecules, nuclei exhibit a rich and complicated spectra of excited states, and these can tell us about the nuclear forces holding the nucleus together.



Similar to how we studied the atomic spectrum of hydrogen!

Another example of a nuclear reaction, this time using a beam of α particles (i.e. ^{4}He nuclei) hitting a ^{12}C target to make excited ^{16}O nuclei via a nuclear fusion reaction:

$^{4}He + {}^{12}C \rightarrow {}^{16}O^{*}$ * denotes excited state of the nucleus

$$^{16}O^{*} \rightarrow {}^{16}O + \gamma$$

This is typically written $^{12}C\ (\alpha\ ,\gamma)\ {}^{16}O$

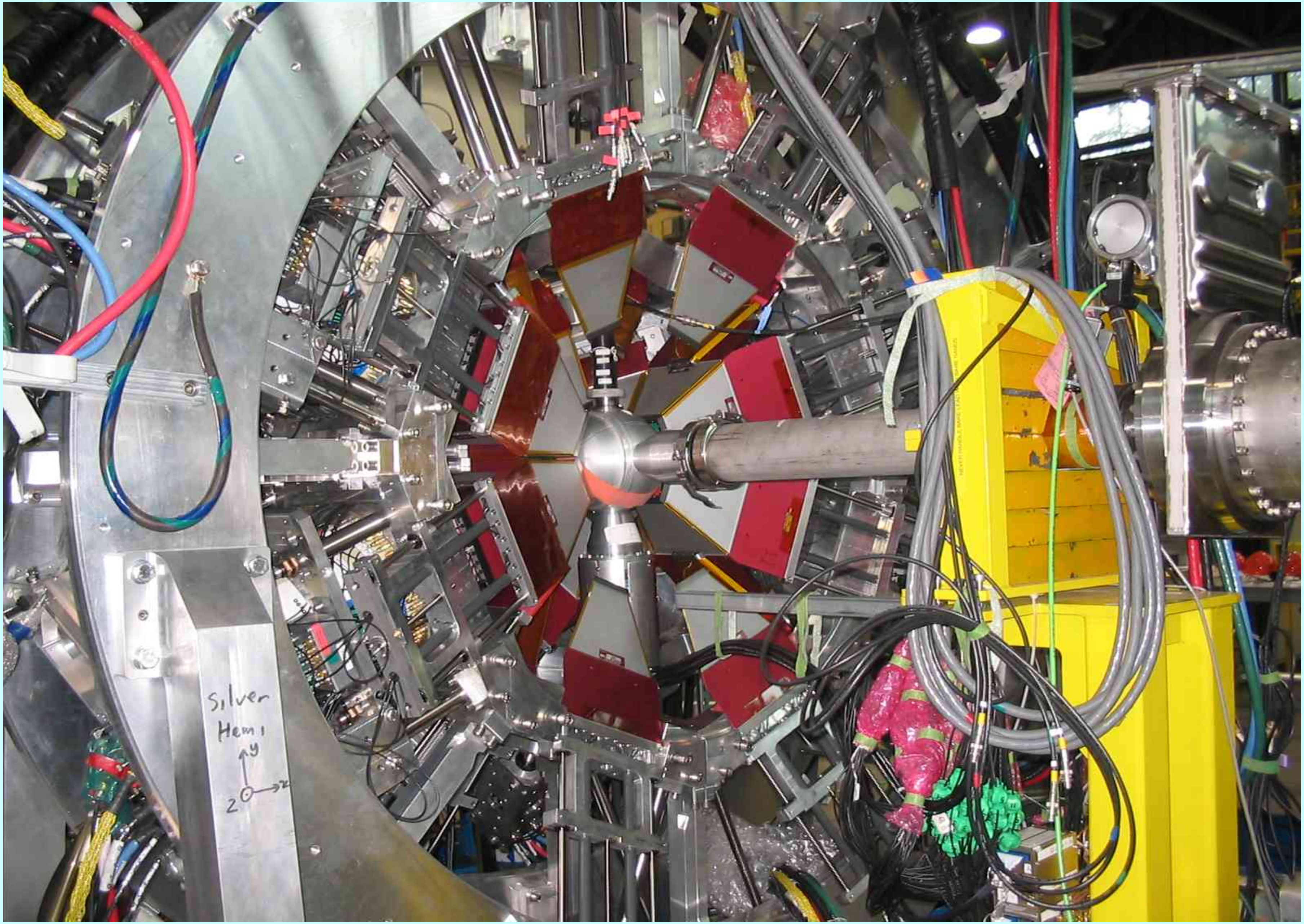
target

projectile

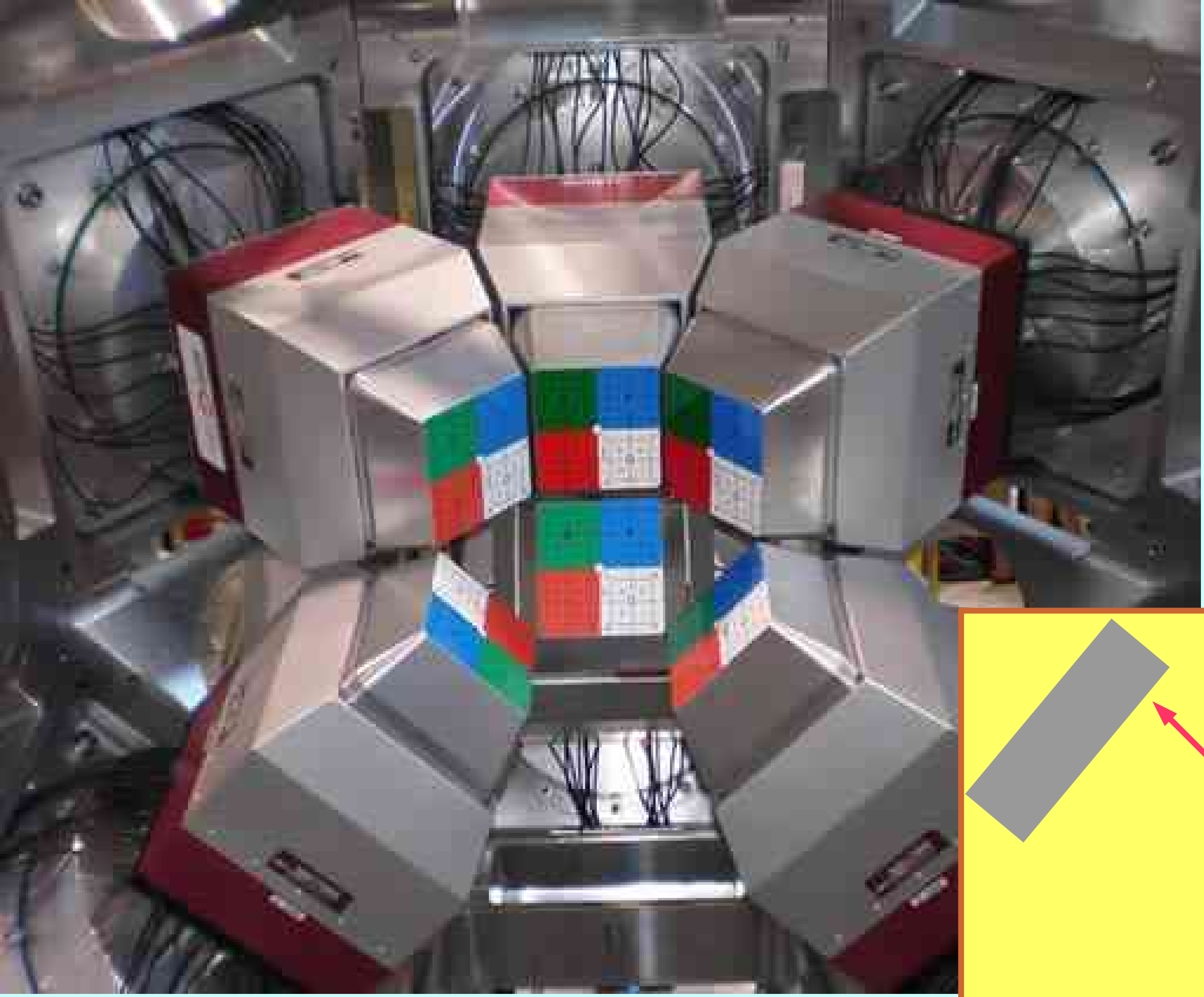
detected reaction product

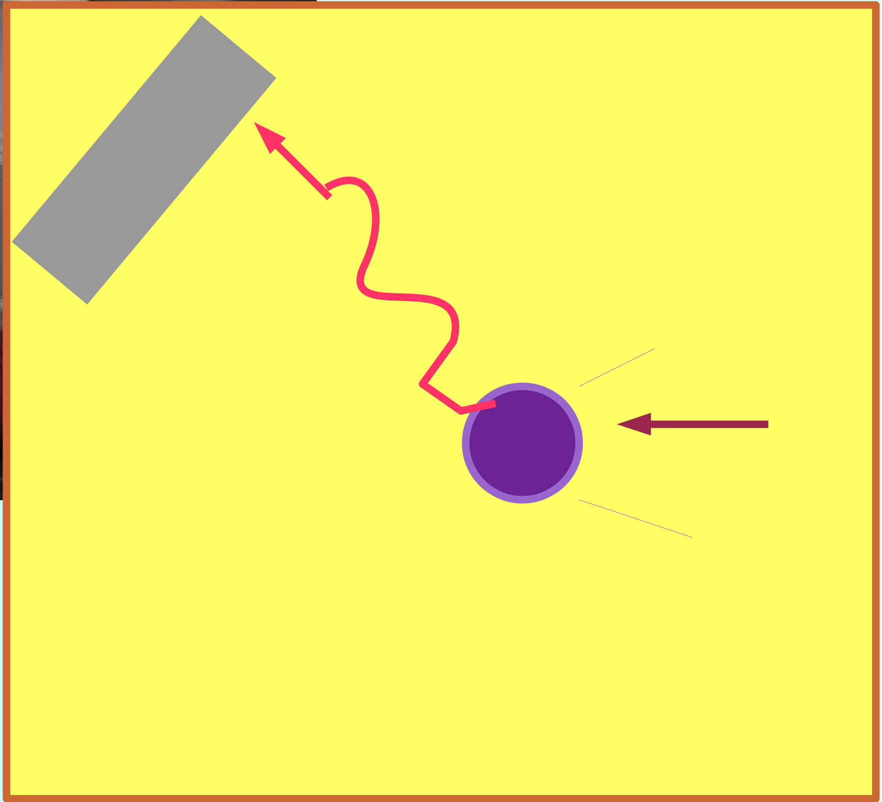
undetected final nucleus

A lot more on nuclear reactions in Barry Davids' lecture!



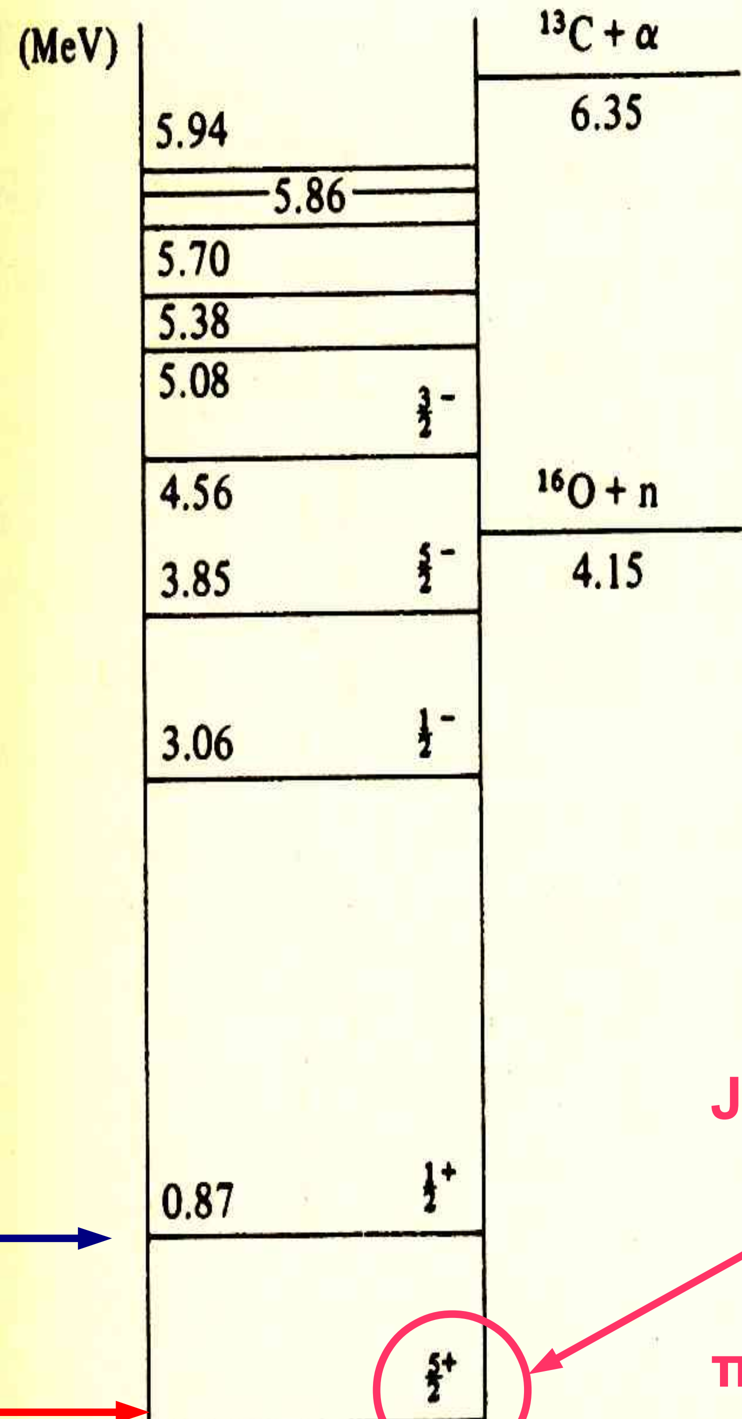
Tigress gamma ray spectrometer now under construction in ISAC-II experimental hall.





Tigress is position-sensitive to allow precise compensation for Doppler shift due to motion of the recoiling nucleus.

Example:
Energy levels of ^{17}O
from Cottingham & Greenwood, Intro to Nuclear Physics



1[st] excited state 0.87 MeV above ground state

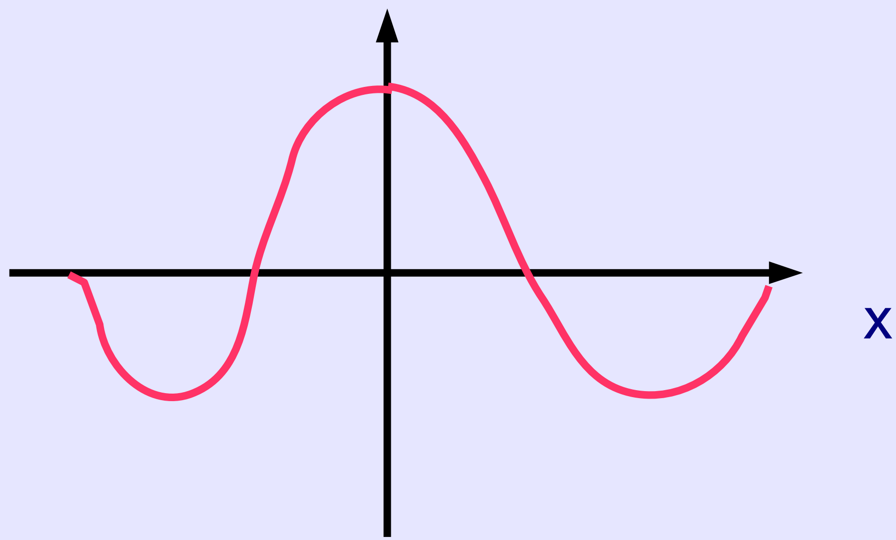
ground state

J^π where
J = spin of state in units of $h/2\pi$
π = parity (+1 or -1)

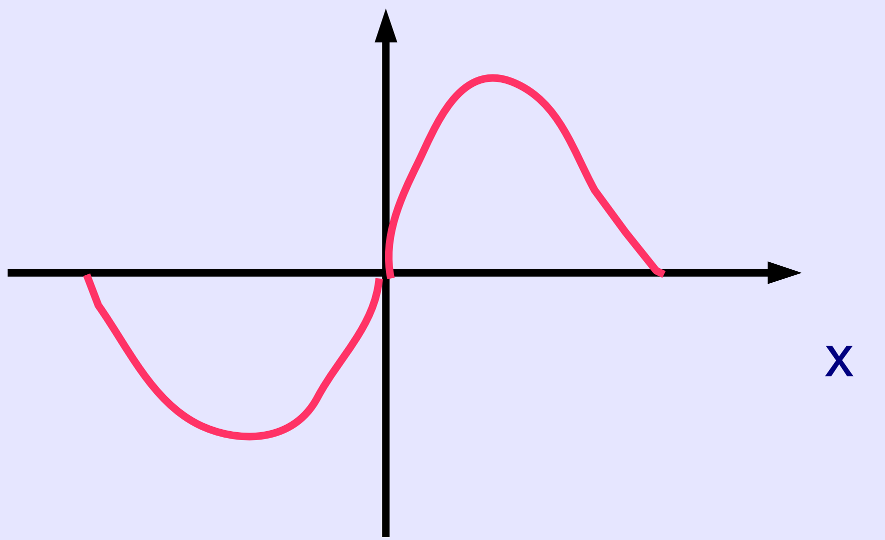
Parity: tells whether the wavefunction is even or odd when $x \rightarrow -x$

If $\psi(-x) = \psi(x)$ then parity $\pi=+1$ (even parity)

If $\psi(-x) = -\psi(x)$ then parity $\pi= -1$ (odd parity)



even parity wavefcn



odd parity wavefcn

Nuclear wavefunctions are either pure even or pure odd parity – mixed parity is forbidden, except for the tiny effects of the weak nuclear force – this is a result of "the conservation of parity" (later lecture on symmetries).

Important features of the Nuclear Binding Force

1. Short-ranged (a few fm)

2. Attractive at the distances > 0.6 fm – that's what binds the nucleons together in a nucleus.

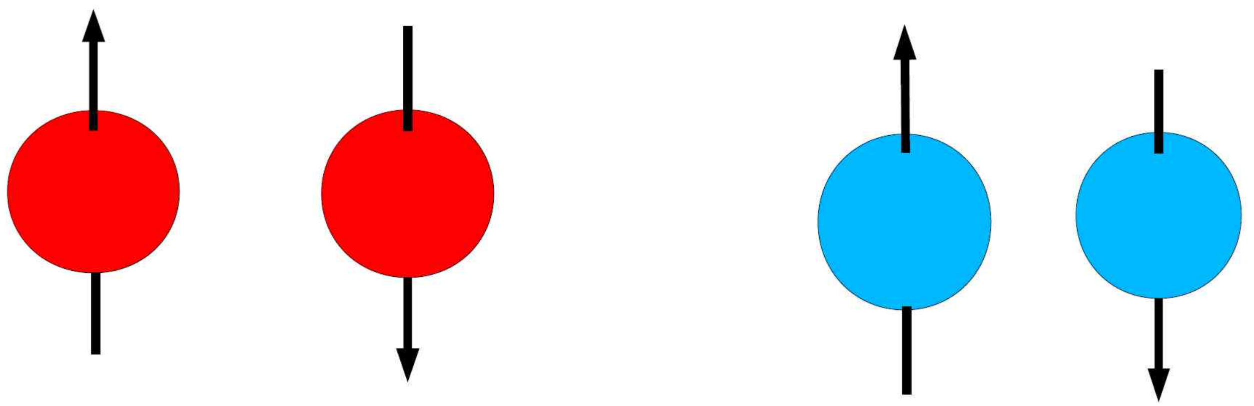
3. Strongly repulsive at short distances of < 0.5 fm – that's why nuclear matter is highly incompressible, and this causes the outward “bounce” of the shock wave in a core-collapse supernova.

4. Strong spin dependence – quite unlike electromagnetic interactions in an atom or molecule.

5. Doesn't distinguish between p-p, p-n or n-n, as long as they are in the same spin orientation.

4. Strong spin dependence

Recall that protons and neutrons are spin-1/2 particles, i.e. they have intrinsic angular momentum ½ in units of $h/2\pi$. Relative to some direction z, the proton's spin axis is quantized to be in one of two possible orientations: either parallel or antiparallel to z.

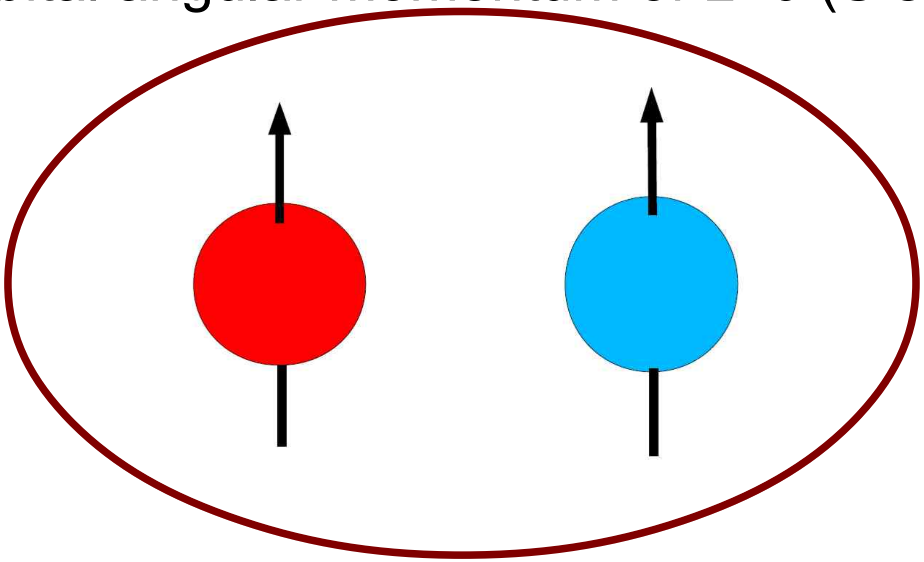


proton spin up | **proton spin down** | **neutron spin up** | **neutron spin down**

The force between two nucleons depends strongly on their relative spin orientations, i.e. spins parallel or spins anti-parallel

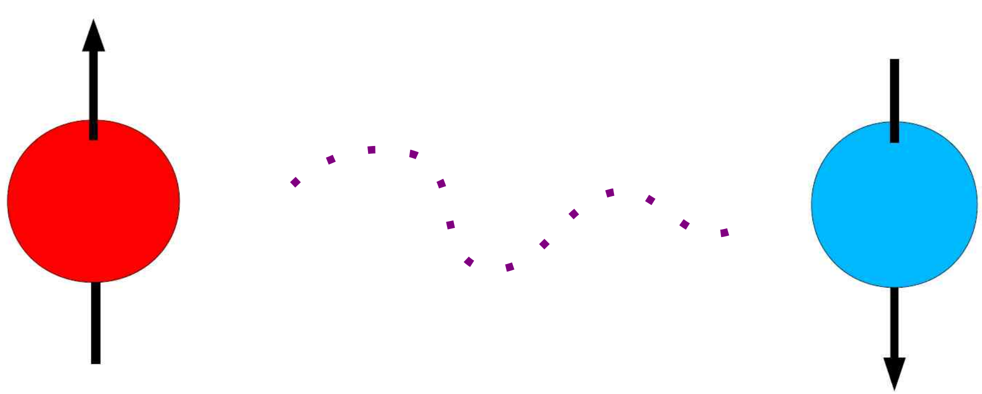
Deuteron (^{2}H nucleus) J=1

consists of a proton and neutron with parallel spins, and relative orbital angular momentum of L=0 (S state), i.e. no orbital motion

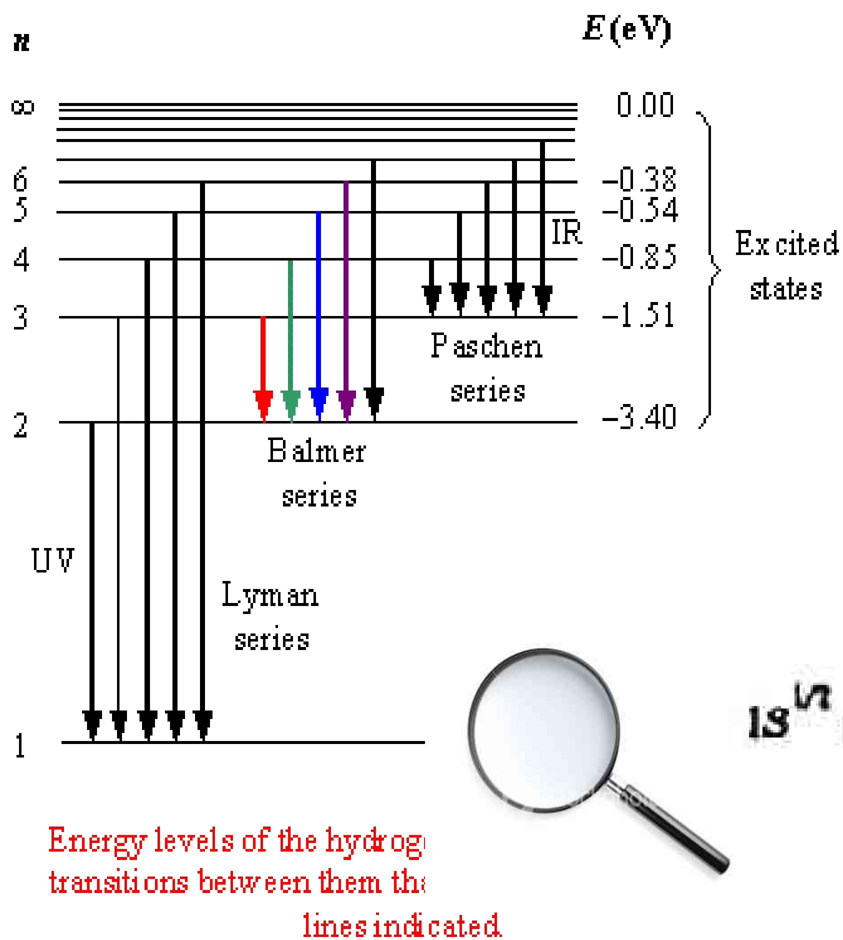


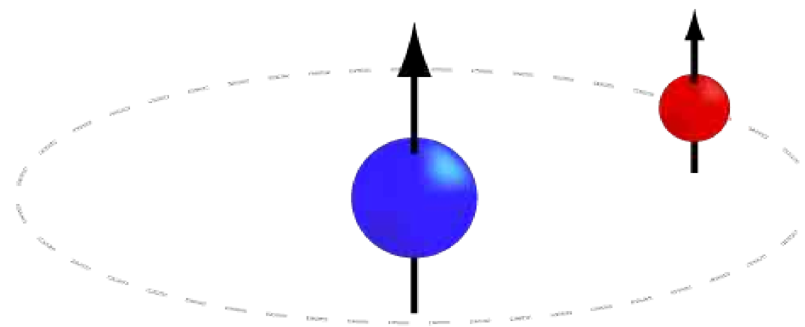
binding energy = 2.2 MeV

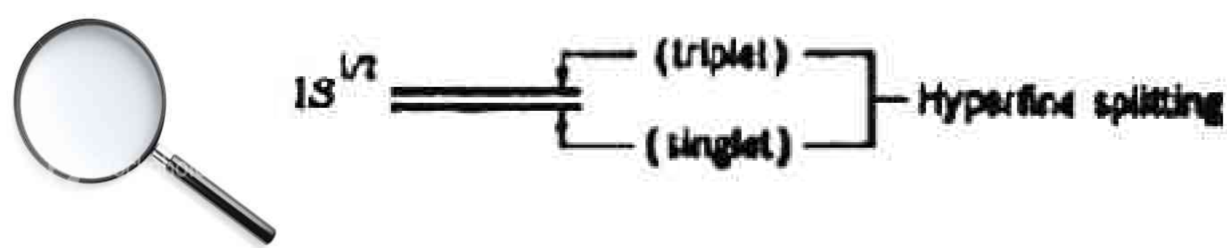
If we try to assemble a proton and neutron with anti-parallel spins, the system will not bind together – it instantly falls apart.



This is quite unlike the H atom, where the spin parallel and spin anti-parallel orientations result in a **tiny** splitting of the 1s level -- the origin of the 21 cm radio emission that radio astronomers use to map out hydrogen in the galaxy.

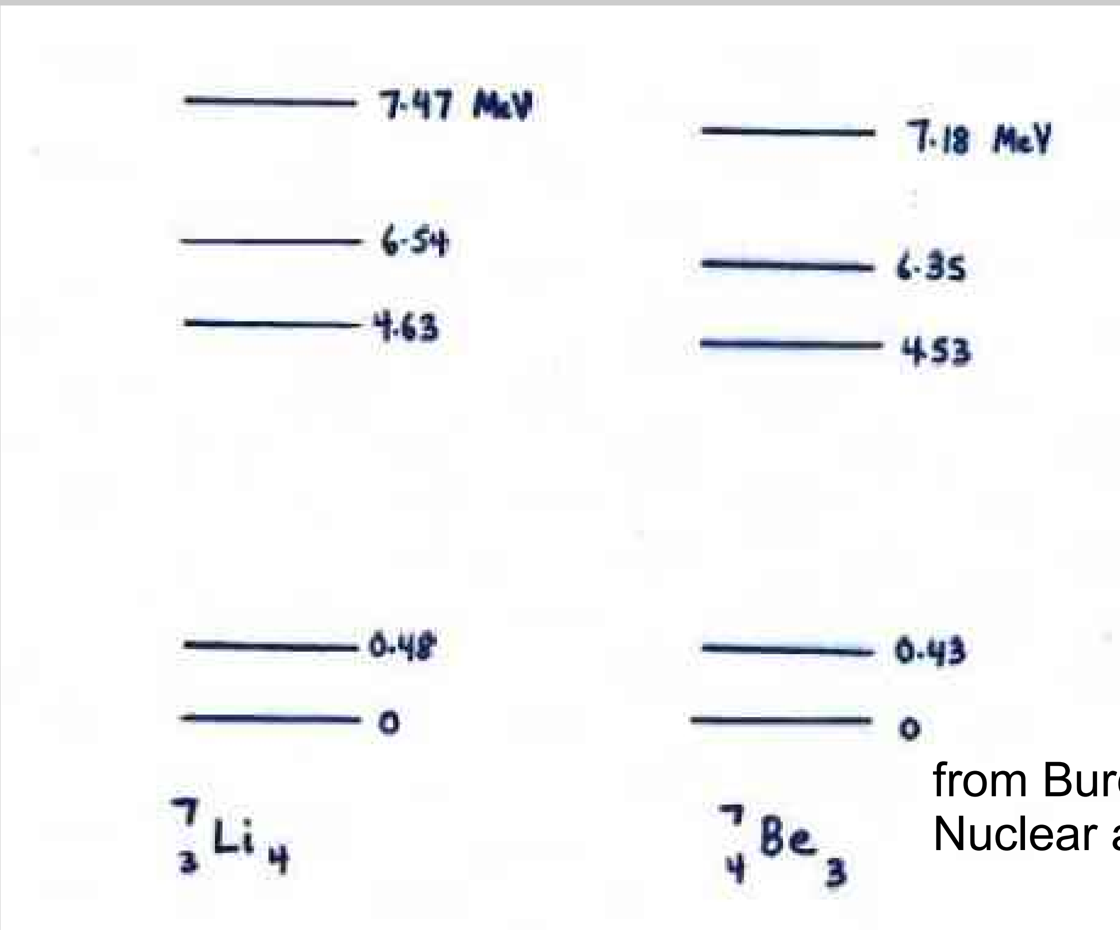






Nuclear forces don't distinguish between protons and neutrons (neglecting the Coulomb interaction) as long as the two nucleons involved are in the same spin orientation.

Evidence: consider the energy levels of ^{7}Li (3 protons, 4 neutrons) and ^{7}Be (4 protons, 3 neutrons). After subtracting off the effect of the Coulomb interaction, the energy levels are almost identifical. It doesn't matter if you switch neutrons ↔ protons ! "CHARGE INDEPENDENCE OF NUCLEAR FORCES"



from Burcham and Joos,
Nuclear and Particle Physics.

If the nuclear forces don't distinguish between protons and neutrons, then in some sense, we can regard protons and neutrons as two manifestations of the same particle.

e.g. electrons are spin-1/2 particles ($S=1/2$)
with two possible spin states $S_z = +1/2$ or $-1/2$
that we call "spin up" and "spin down".

By analogy:

nucleons are isospin-1/2 particles ($I=1/2$)
with two possible isospin states $I_3=+1/2$ (proton) or $I_3=-1/2$(neutron)

i.e. "up" and "down" in "isospin space"

(This is the notation used in particle physics; nuclear physicists typically use $T=1/2$, and assign neutrons to be $T_3=+1/2$ and protons to be $T_3= -1/2$).

Yukawa Hypothesis ; Mesons

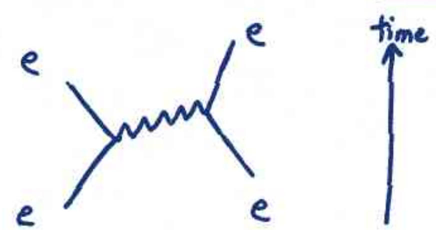
By the early 1930's it was known that nuclear forces had a range of ≈ 2 fm.

Analogy with electromagnetism : two charged bodies do not interact by "action at a distance", but each body is the source of an electric <u>field</u> which interacts with the other body.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{e}{r^2} = K\frac{e}{r^2}$$

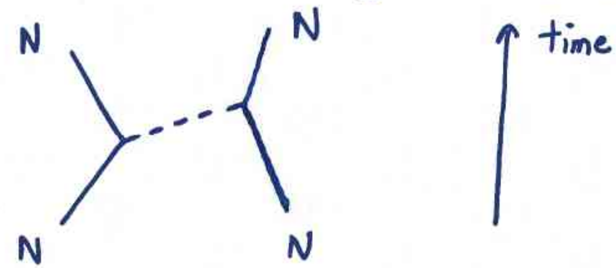
Potential $\phi = \frac{Ke}{r}$

The EM field is quantized, and the quanta of the EM field are <u>photons</u>. Charged particles interact by exchanging photons. Since photons are massless, the EM field has ∞ range.



"Feynman diagrams"

In 1935 Yukawa postulated that the strong nuclear force is carried by quanta called "mesons". Nucleons interact by exchanging mesons.



Suppose that mesons have mass m.
Then creating a meson would violate energy conservation, but by the uncertainty principle, we can do this for a short time $\Delta t \lesssim \dfrac{\hbar}{\Delta E} = \dfrac{\hbar}{mc^2}$

In this time Δt, the meson can travel a distance of, at most, $R = c\,\Delta t = \dfrac{\hbar c}{mc^2}$

Using $\hbar c = 197.329$ MeV-fm (back cover of Krane)
and knowing $R \sim 2$ fm

we get $\quad 2 \text{ fm} = \dfrac{197.3 \text{ MeV-fm}}{mc^2}$

$$mc^2 \sim 100 \text{ MeV}.$$

The quanta of the strong force have mass ~ 100 MeV, which is between electrons and nucleons, so they were called mesons ("middle weight").

The exchange of mesons of mass m gives rise to a potential

$$\Phi = g\,\frac{e^{-\alpha r}}{r} \qquad \alpha = \frac{mc}{\hbar}$$

This is called a Yukawa potential

In the limit $m \to 0$, we get back the familiar Coulomb potential

$$\Phi = g\,\frac{1}{r} \equiv \frac{e}{4\pi\epsilon_0}\,\frac{1}{r}$$

So we can think of g as the "strong charge" giving rise to a meson potential $g\,\frac{e^{-\alpha r}}{r}$ in the same way that e is the "electric charge" giving rise to an electric potential $\frac{e}{4\pi\epsilon_0}\frac{1}{r}$

The range of the Yukawa potential is approximately $1/\alpha = \frac{\hbar}{mc} = \frac{\hbar c}{mc^2}$

Exactly what we got on the previous slide, using the uncertainty principle argument.

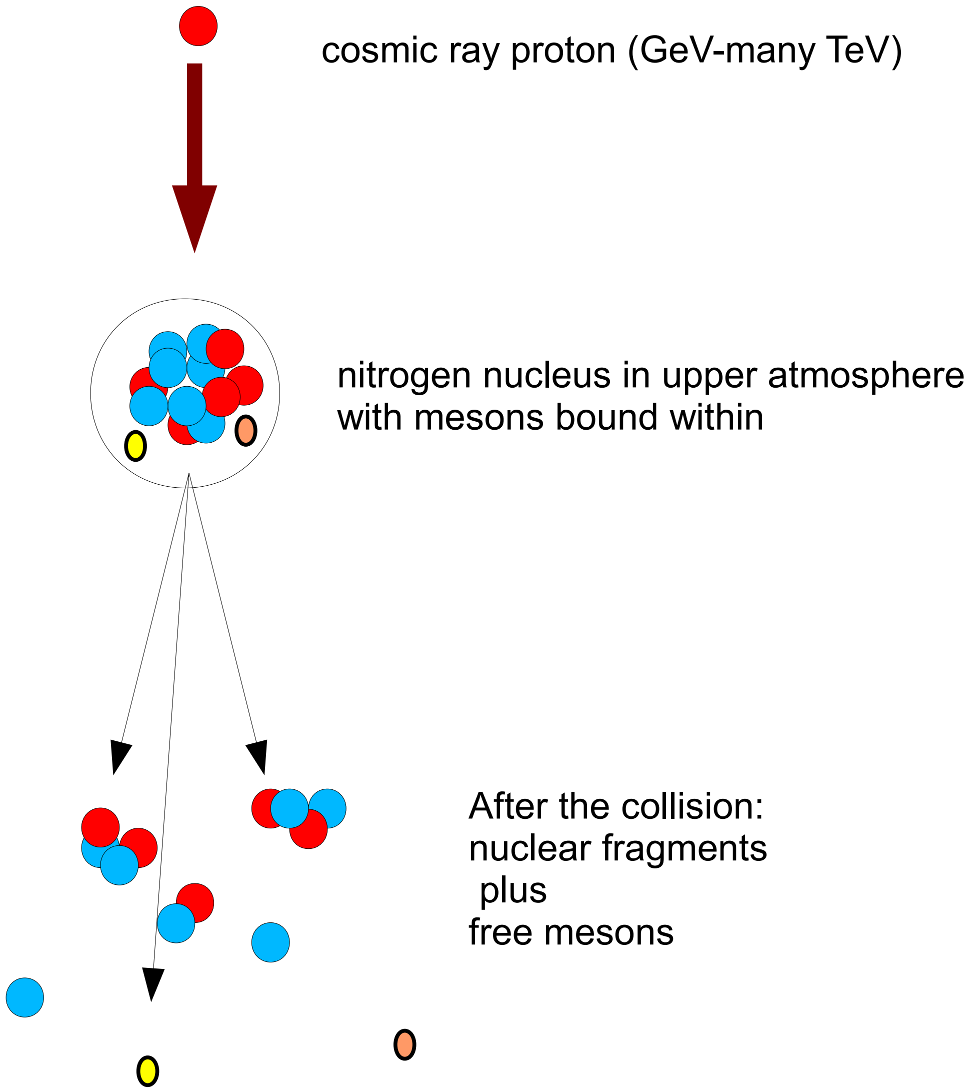
Heavier meson means shorter range

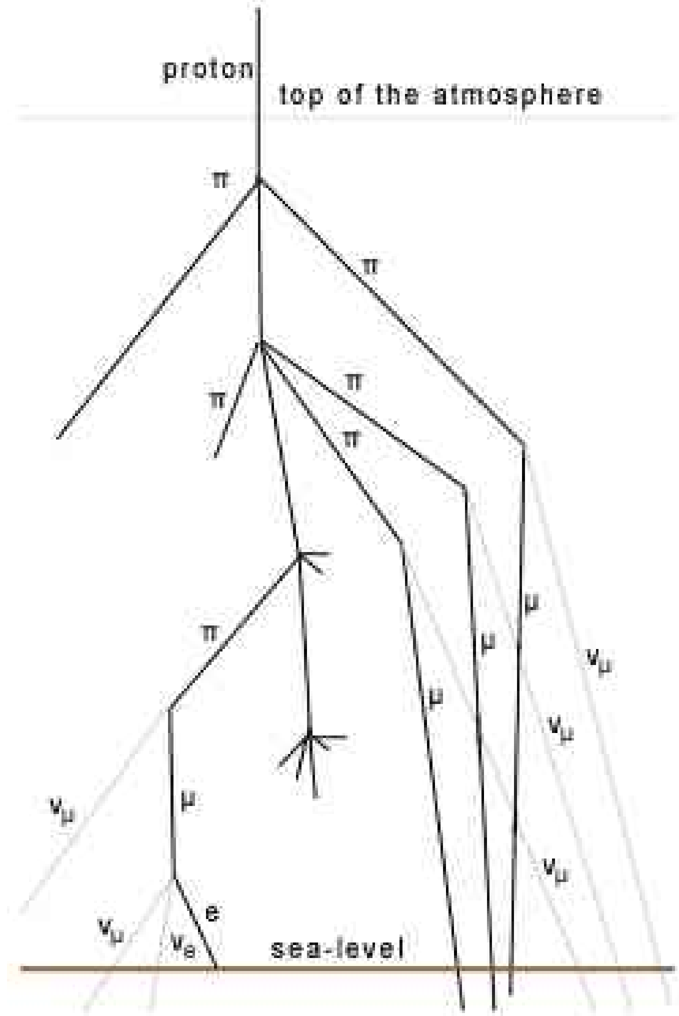
1947: Search for Yukawa particle by exposing stacks of photographic emulsions to cosmic rays at high altitude

Pic du Midi observatory in French Pyrenees

High Altitude Balloons

from lecture "Discovery of the Pion" by Anton Kapliy





picture from bramsci.livejournal.com

First pion

Nuclear capture of pion

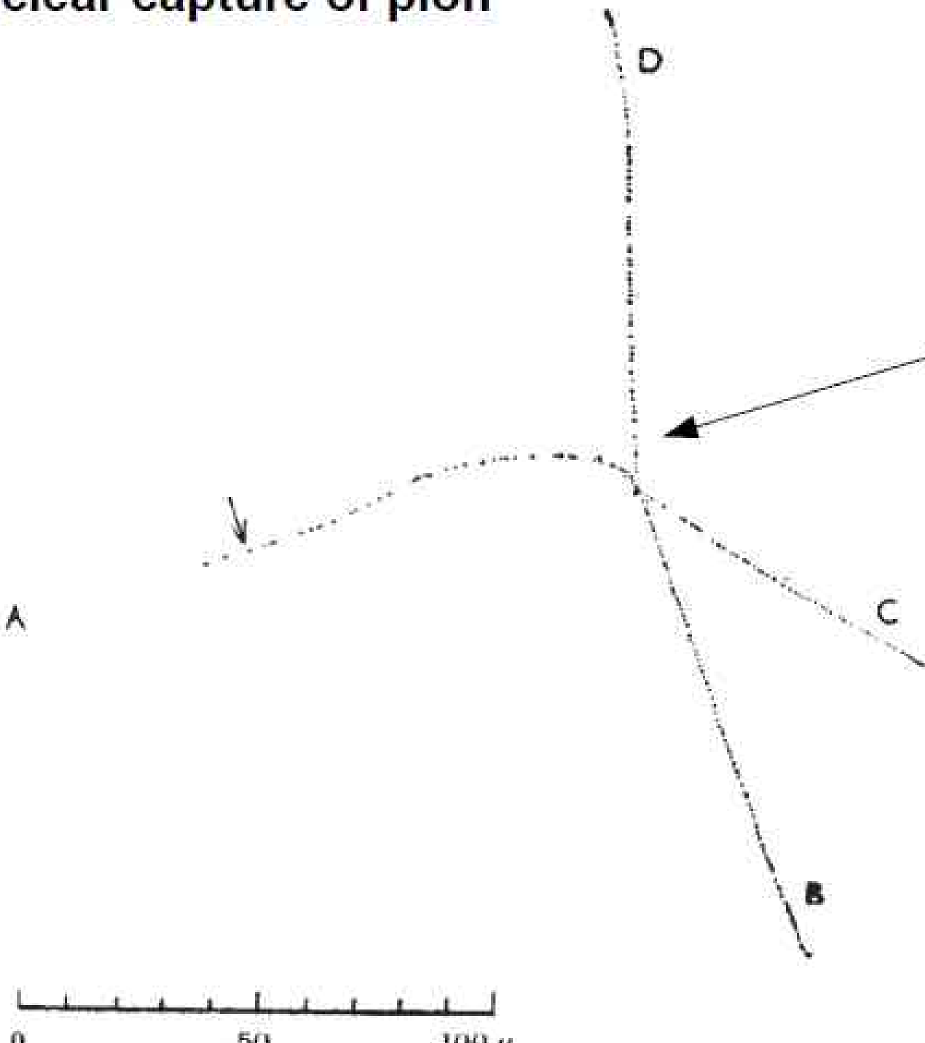
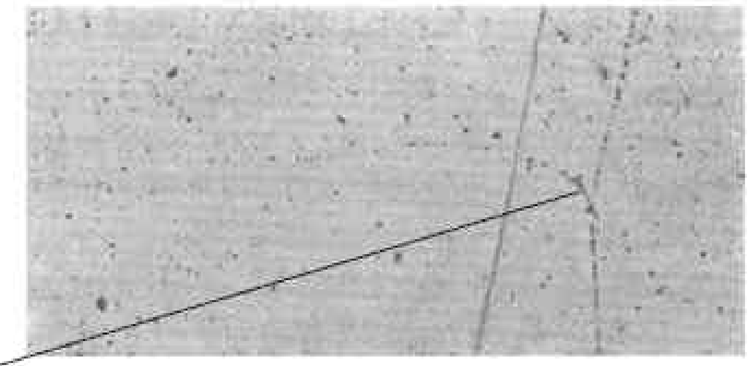


Fig. 1 *b*. TRACE OF COMPLETE STAR ON SCREEN OF PROJECTION MICROSCOPE, SHOWING PROJECTION OF THE TRACES IN THE PLANE OF THE EMULSION. TRACK *A* CANNOT BE TRACED WITH CERTAINTY BEYOND THE ARROW



g. 1 *a*. PHOTOMICROGRAPH OF CENTRE OF STAR, SHOWING TRACK OF ESON PRODUCING DISINTEGRATION. (LEITZ 2 MM. OIL-IMMERSION OBJECTIVE. × 500)

- A is the new meson
- B,D,C are likely protons
- Track C goes into the page

Why A is a new meson:
electron: range too large
proton: scattering too large
muon: frequent nuclear interaction

(Jan 1947, observed by D. Perkins)

This new particle DOES interact strongly with the nucleus! When it gets captured by the nucleus, the nucleus swallows an energy equivalent to the pion mass (~139 MeV) and blows up, emitting nuclear fragments (thus the energetic protons emerging from the end of the pion track on the previous page).

The pion comes in 3 charge states (π^+ , π^0 , π^-) and are short-lived

$\pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}$ τ = 26 nsec (2.6 x 10^{-8} sec)

$\pi^0 \rightarrow \gamma + \gamma$ $\tau = 10^{-16}$ sec

There are numerous heavier mesons, which live even shorter lives

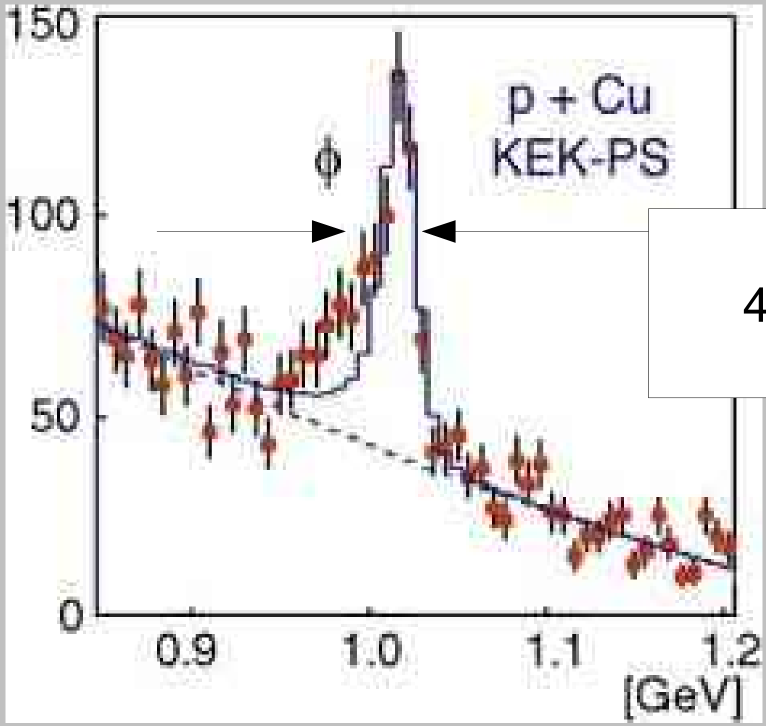
η	mass=548 MeV	$\tau = 5 \times 10^{-19}$ sec
ρ	mass=775 MeV	$\tau = 4 \times 10^{-24}$ sec
ω	mass=783 MeV	$\tau = 8 \times 10^{-23}$ sec
φ	mass=1020 MeV	$\tau = 1.6 \times 10^{-22}$ sec

From the Heisenberg uncertainty principle

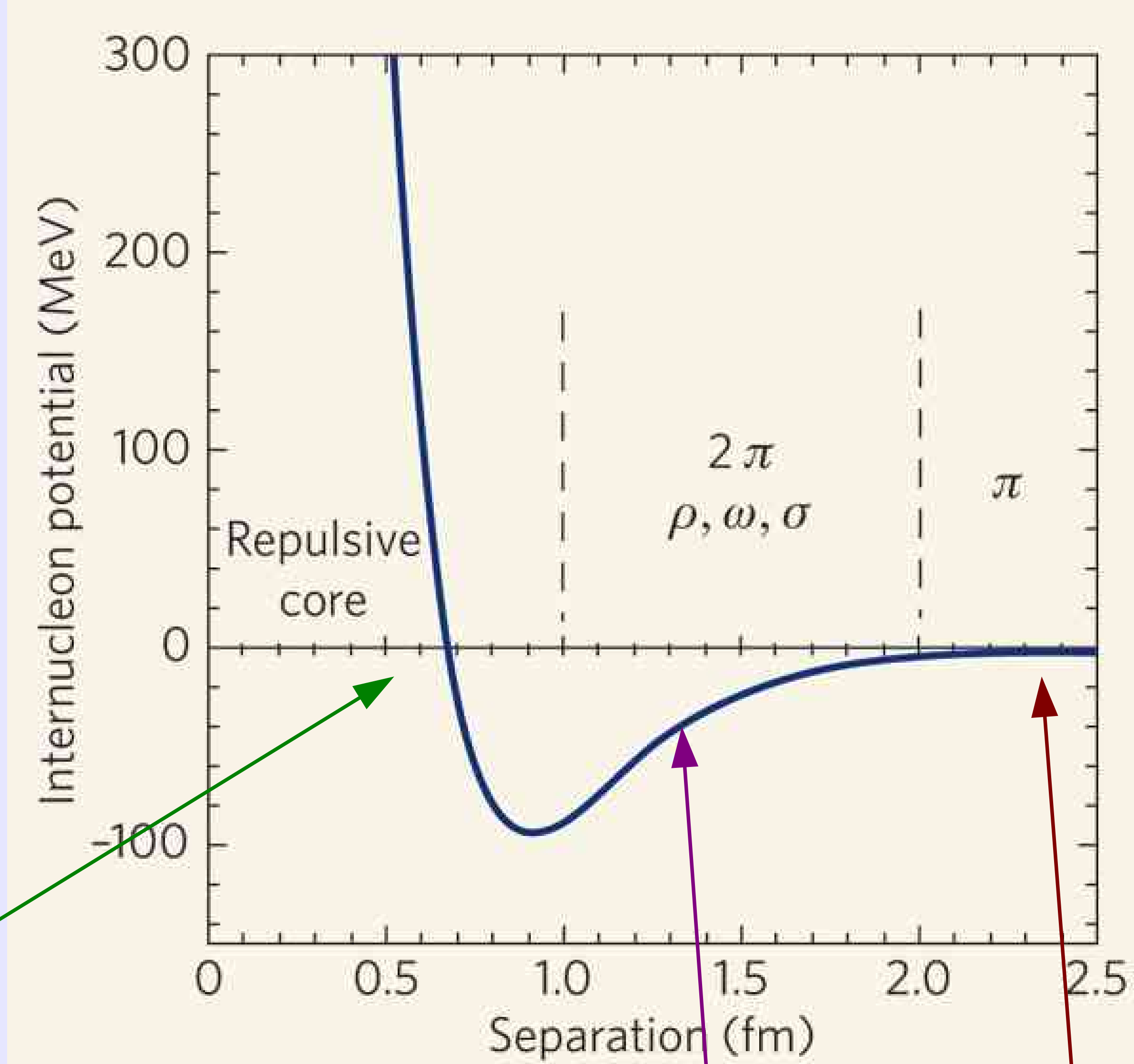
$\Delta E \, . \, \Delta t > h/2\pi$

the shorter the lifetime Δt of a quantum state (such as a meson) the greater the uncertainty in its mass or energy ΔE.

So when these short-lived mesons are produced in a high energy particle collision, and you try to measure their mass, they show a width due to the uncertainty principle.



4.26 MeV wide



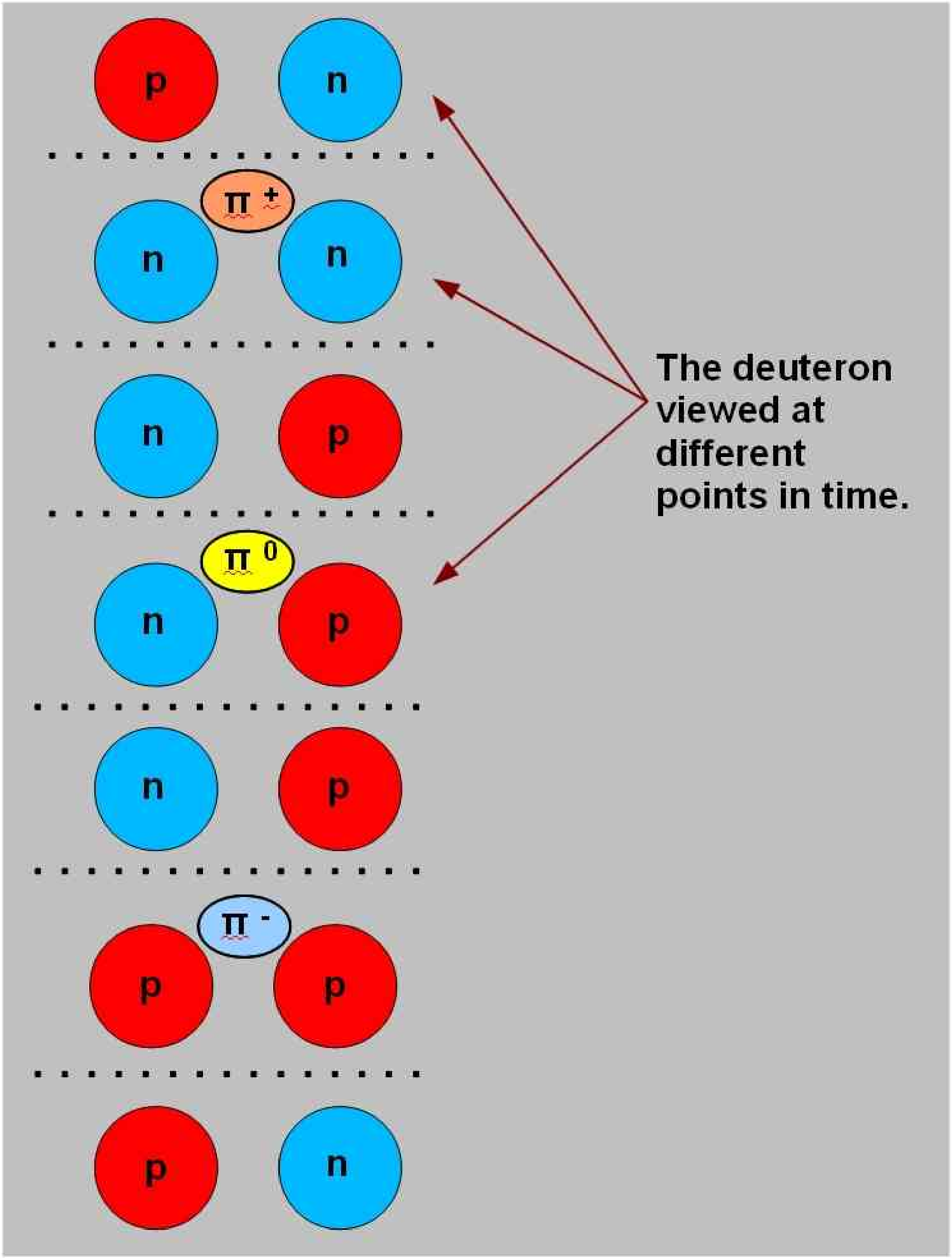
overlap of
3-quark bags;
complicated
short-range
behaviour

two pion
and
heavy meson
exchange

one pion
exchange

The nucleus is a dynamic object where pions and heavier mesons constantly flit in and out of existence for only as long as permitted by the uncertainty principle.

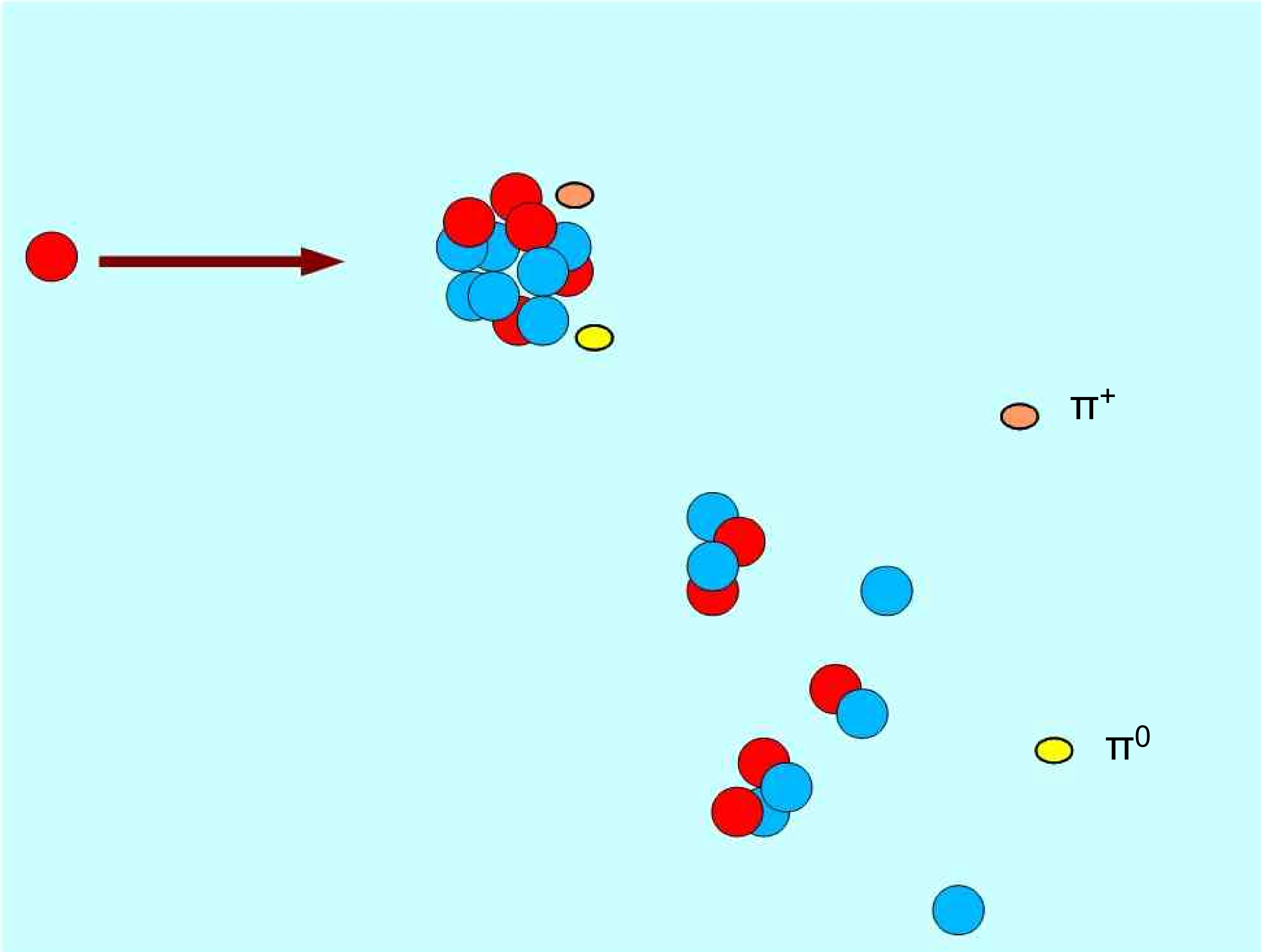
These “temporary” mesons that exist by energy borrowed from the uncertainty principle are called “virtual mesons”.



TRIUMF = TRI University Meson Facility

A cyclotron designed to produce large numbers of π mesons and muons (millions to hundreds of million per sec)

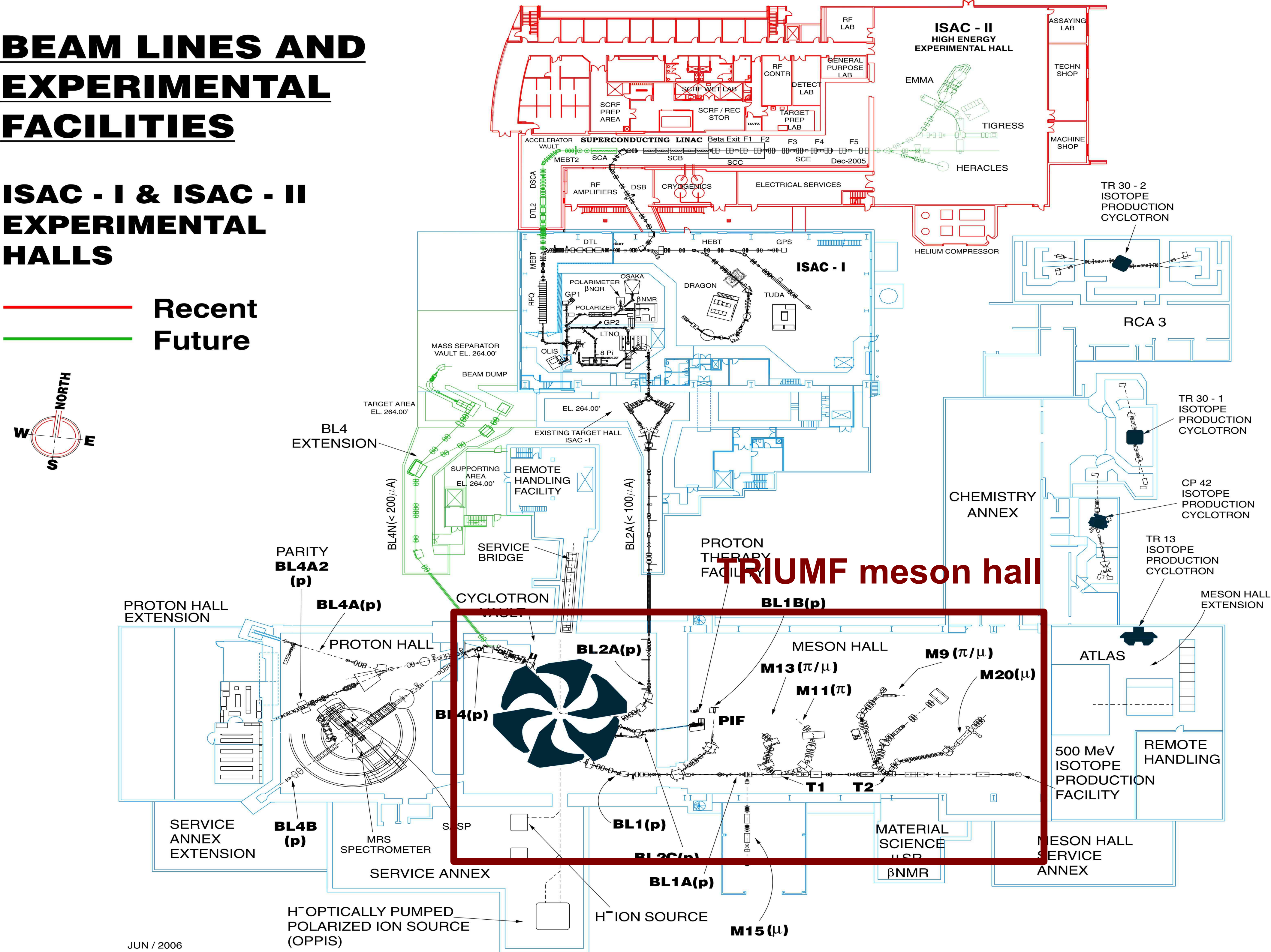
How?
Blast a carbon nucleus with a high energy proton, turn those fleeting virtual π mesons into free particles by supplying enough energy. (no longer restricted by using uncertainty principle to borrow energy for a short period of time).

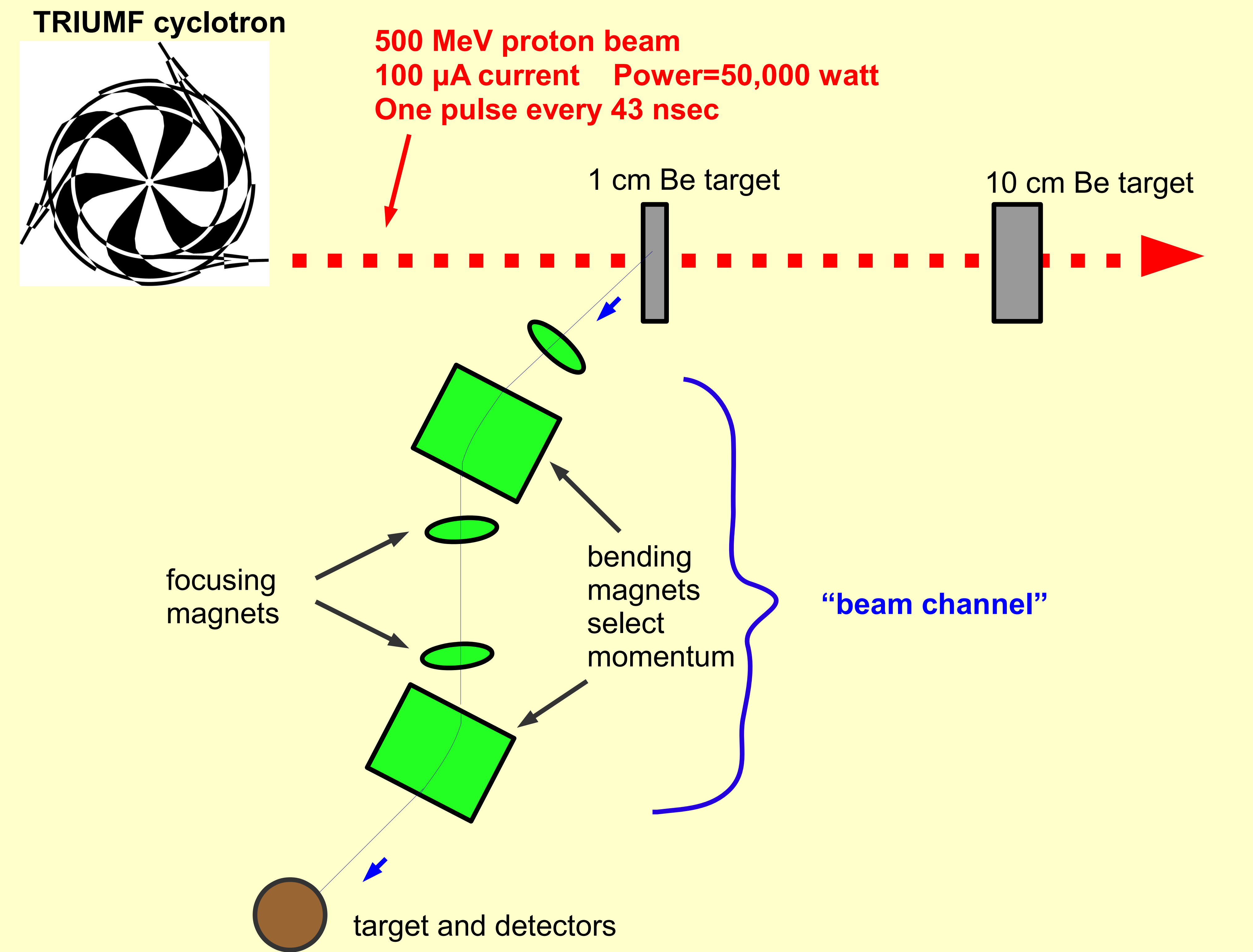


π^+
π^0

BEAM LINES AND EXPERIMENTAL FACILITIES

ISAC - I & ISAC - II EXPERIMENTAL HALLS




TRIUMF cyclotron
500 MeV proton beam
100 μA current Power=50,000 watt
One pulse every 43 nsec
1 cm Be target
10 cm Be target
focusing magnets
bending magnets select momentum
"beam channel"
target and detectors

As free particles

π^+ and π^- have a mean lifetime of 26 nsec in their own rest frame (but longer in the lab frame, because of relativistic time dilation)

and decay into μ^+ and μ^- (plus unobserved neutrinos)

π^0 has a mean lifetime of ~ 10^{-16} sec before decaying into two gamma rays, which hit other material and produce a shower of electrons

μ^+ and μ^- have a mean lifetime of 2.2 μsec before decaying into e^+ and e^- (plus more unobserved neutrinos)

Therefore, what comes out at the end of the beam channel is a generally mixture of charged pions, muons, and electrons/positrons. Special techniques must be used to select one species of particle over another.

The End