Constituents and Shapes of Nuclei and Nucleons

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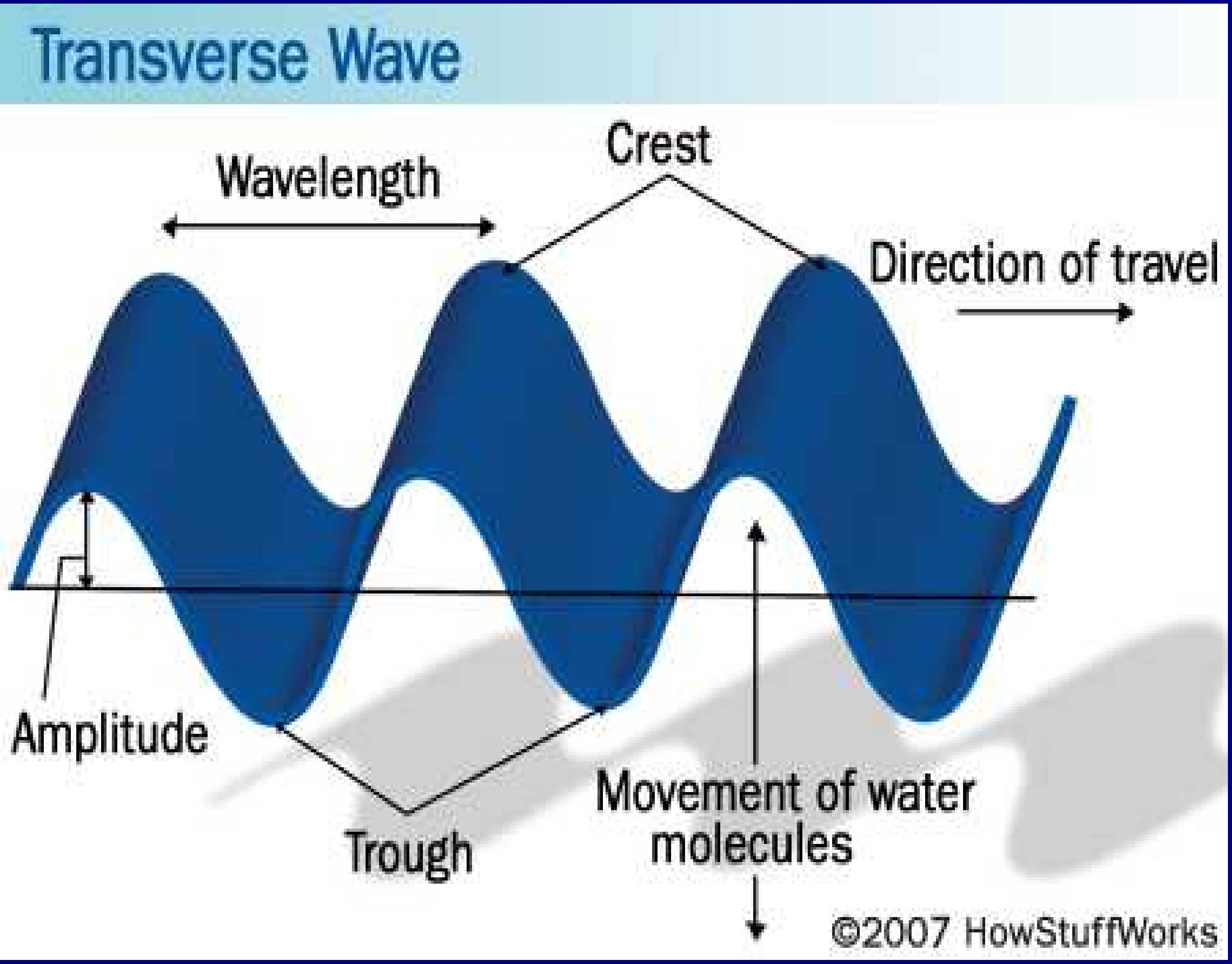
TRIUMF

Quick review of some quantum physics:

1. Basic properties of waves

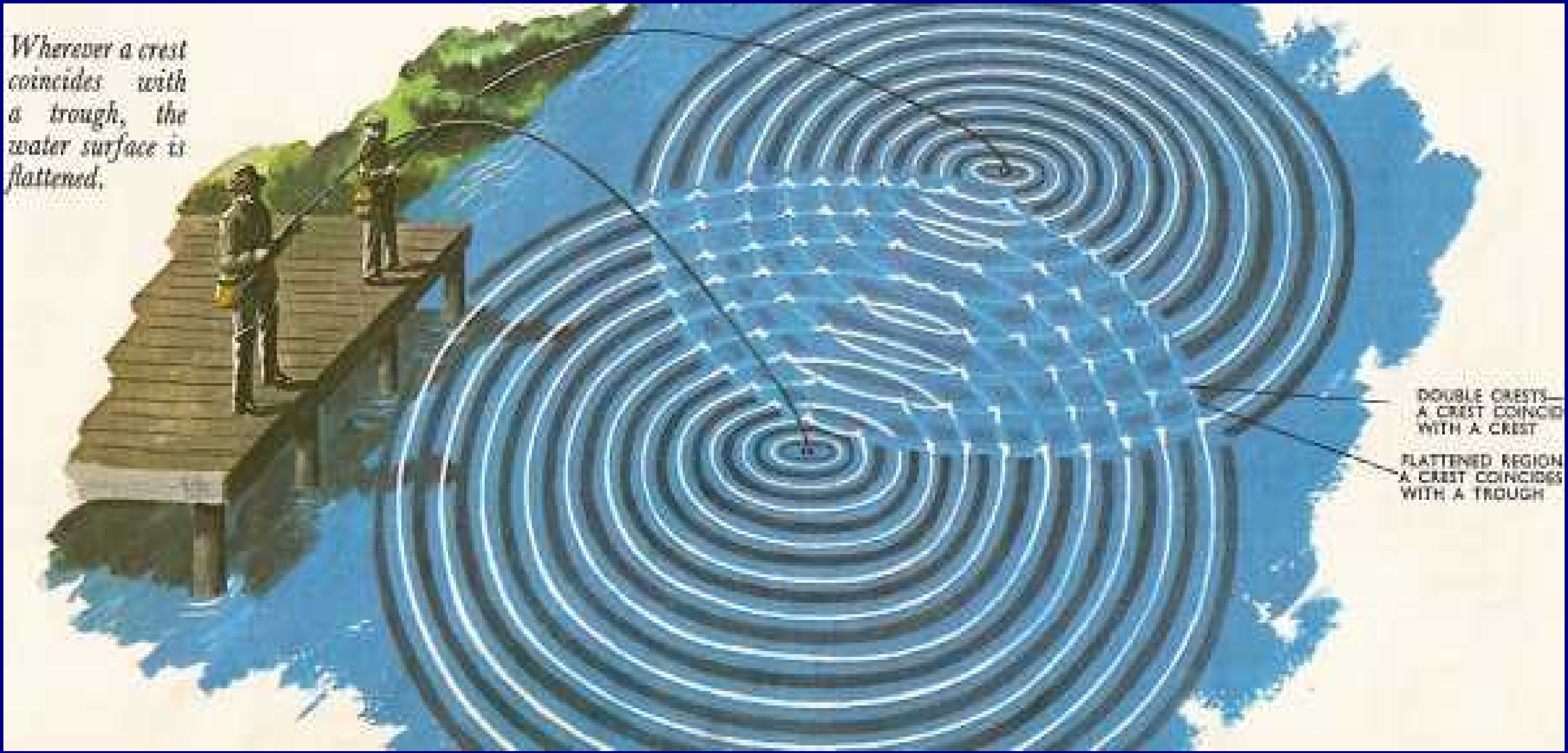


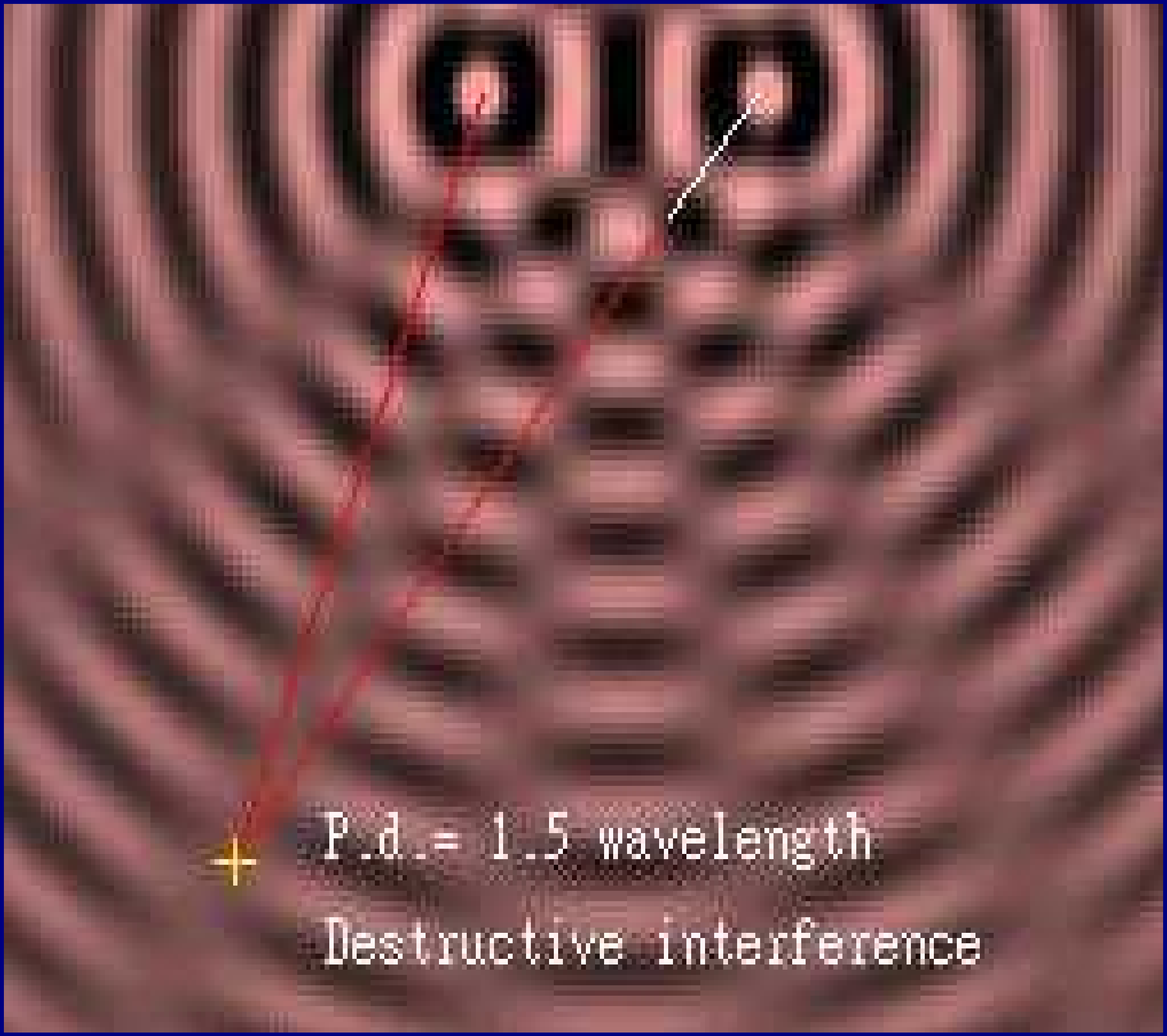
Frequency
= number of crests passing by each second

velocity
= frequency x wavelength

2. Waves can interfere with each other

amplitudes adding up = constructive interference
amplitudes subtracting = destructive interference

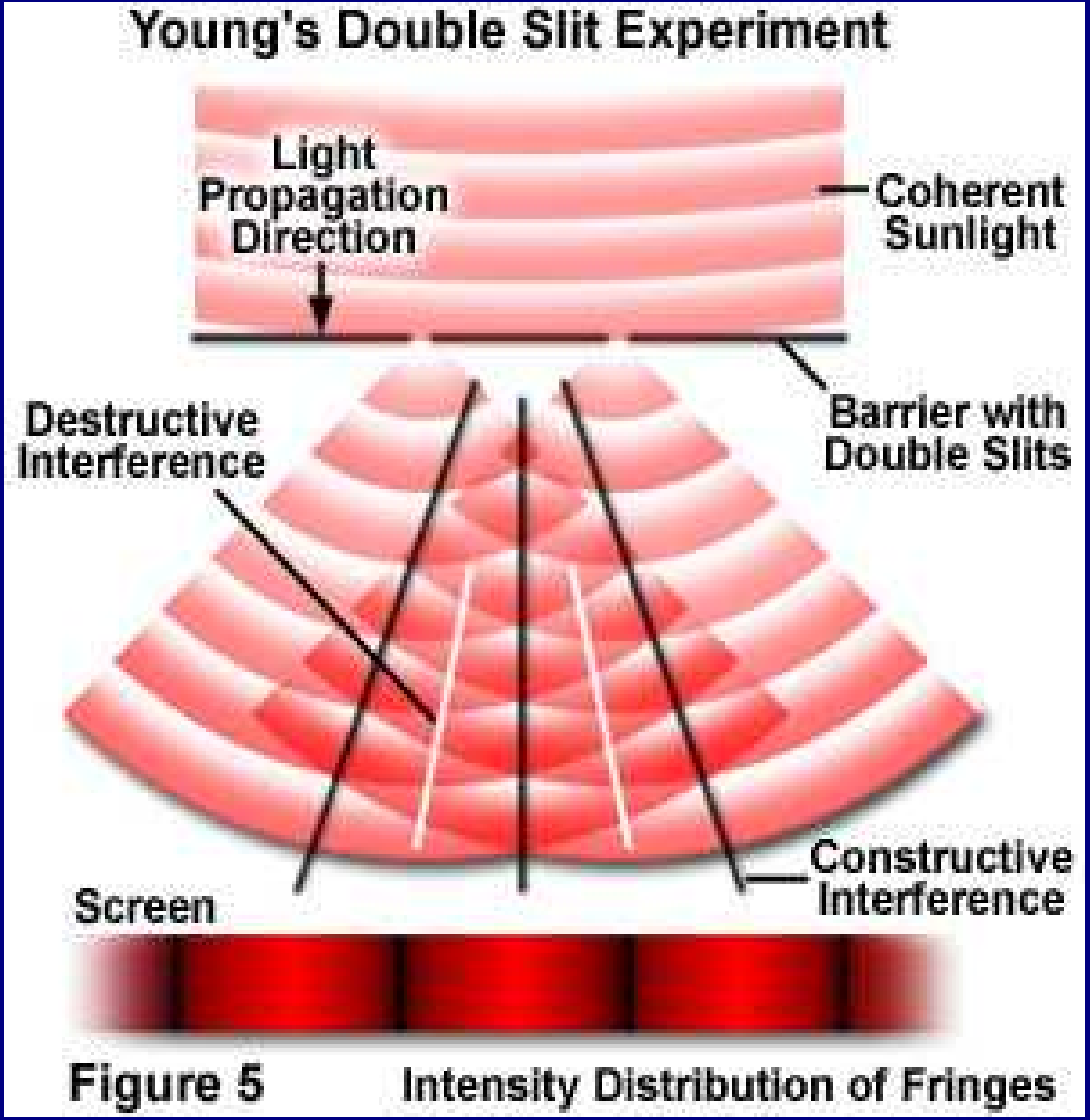




nodal lines of complete destructive interference where crest of one wave cancels out the trough of the second wave

3. Light is a wave, because it shows interference

destructive interference
gives nodal lines
(dark bands of zero light)
just like the nodal lines
in the case of water waves!



4. Light is also a particle !

The photoelectric effect shows that light is not a continuous wave, but the light energy comes in 'packets' of size

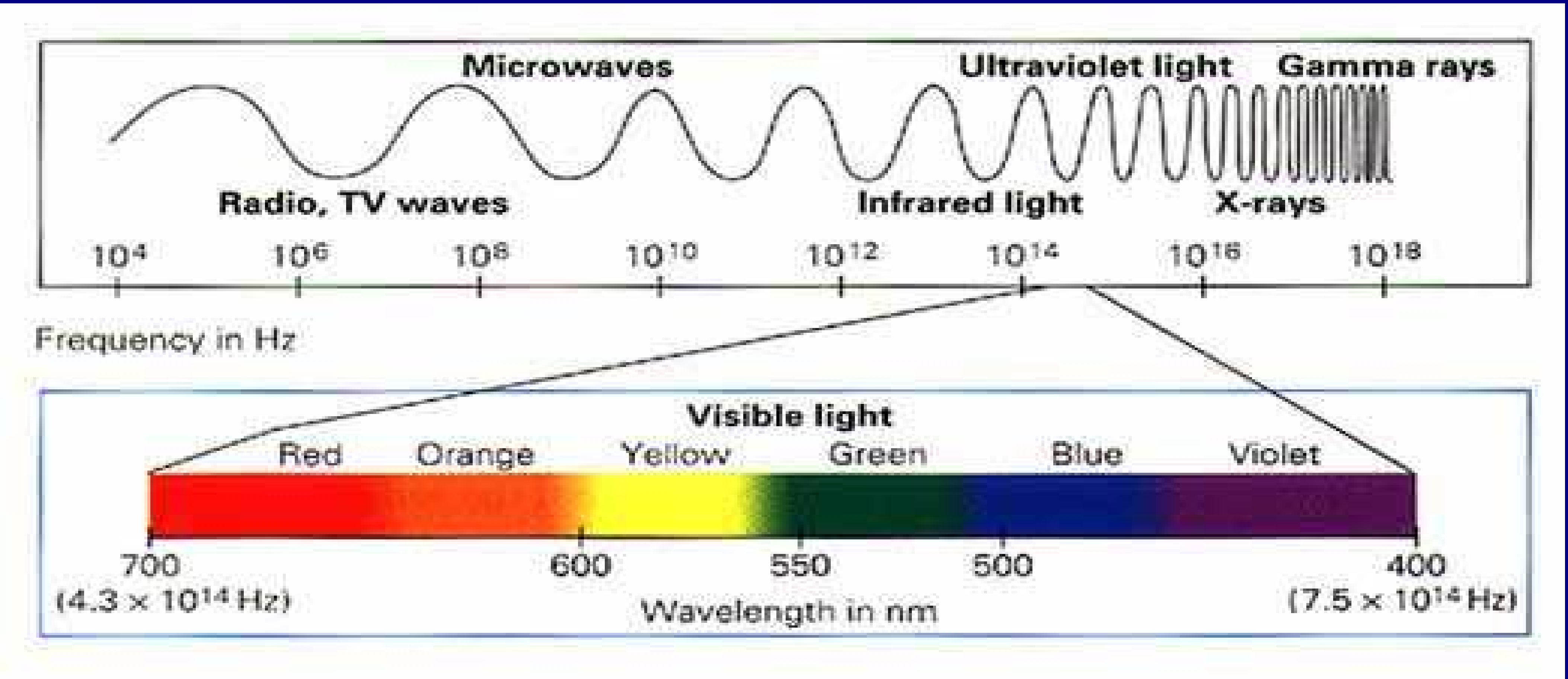
$E = h \cdot f$

where h = Planck's constant
f = frequency of the light

low frequency (red) light = small packets of energy
high frequency (blue) = large packets of energy

$E = 10^{-10}$ eV

$E = 10^{7}$ eV



$E = 0.28$ eV

$E = 0.49$ eV

6. Light is <u>both</u> particle and wave

This is a general property of all matter, not just of light. It is possible to see diffraction patterns made by beams of electrons, neutrons, protons, carbon atoms, and even Buckyballs !

This is the fundamental principle of <u>wave-particle duality</u>

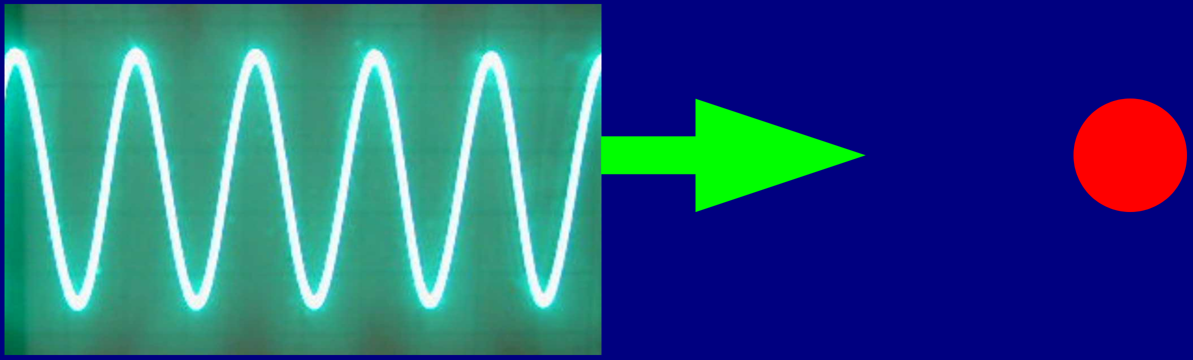
de Broglie:

The wavelength is given by $\lambda = h / p$

where h = Planck's constant
p = momentum of the particle

7. The act of measurement disturbs the system being measured

e.g. measure the position of an electron by observing it with light of wavelength λ, momentum $p = h/\lambda$



When the photon scatters off the electron, it imparts momentum to the electron, so....

Heisenberg uncertainty principle

large momentum p $\rightarrow$ short wavelength λ

$\downarrow$ $\downarrow$

large distubance of electron → precise measurement of position x

$\downarrow$ $\downarrow$

uncertainty of electron momentum $\Delta p \approx p$

uncertainty of electron position $\Delta x \approx \lambda = h/p$

so $\Delta p \cdot \Delta x \approx h$

More precisely $\Delta p \cdot \Delta x \geq h/4\pi$

Similarly for energy and time $\Delta E \cdot \Delta t \geq h/4\pi$

Planck's constant h sets the scale for the size of quantum effects

$$E = h \cdot f$$

$$\lambda = h / p$$

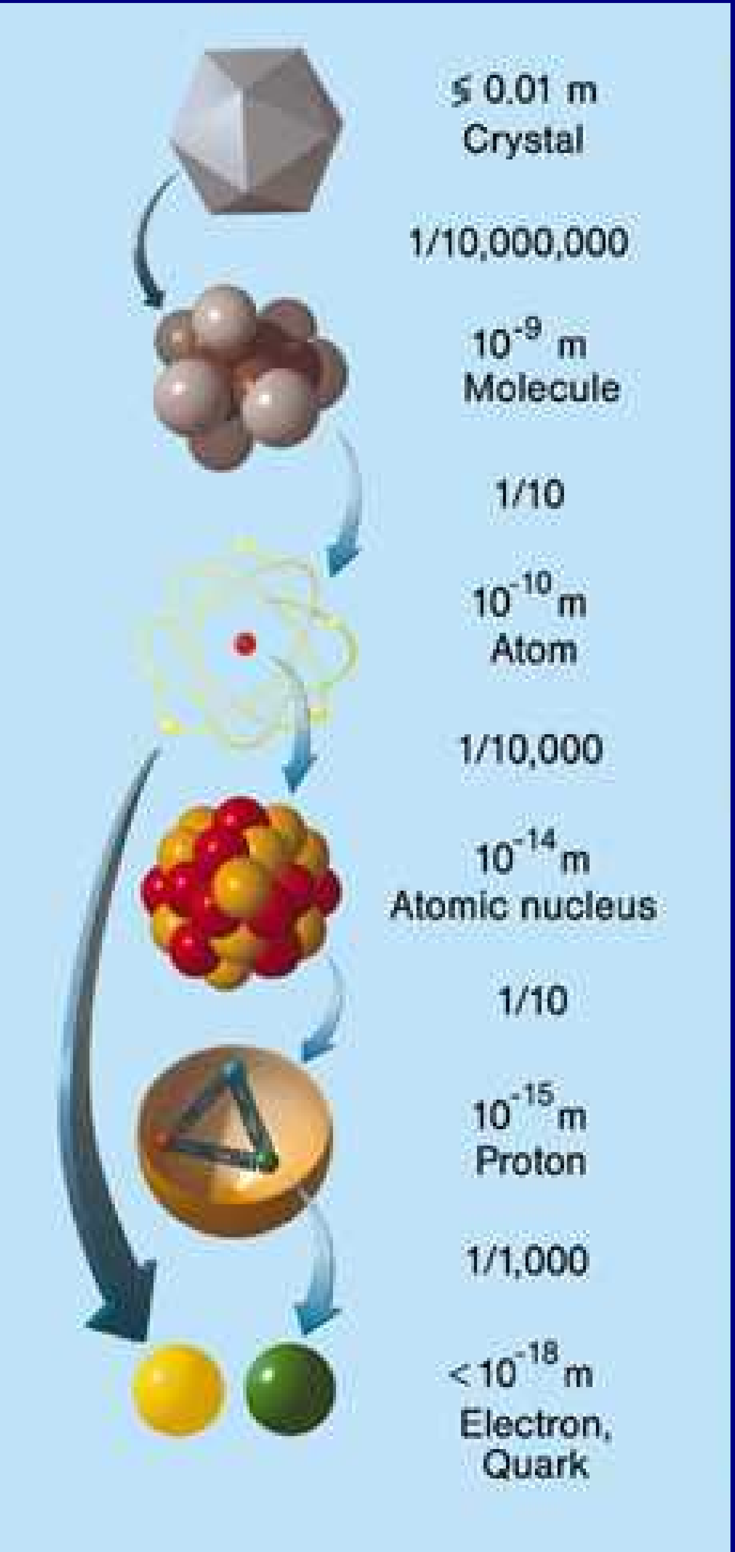
$$\Delta p \cdot \Delta x \geq h/4\pi$$

$$\Delta E \cdot \Delta t \geq h/4\pi$$

$h = 6.627\times10^{-34}$ Joule-sec = 4.13×10^{-15} eV-sec

which is tiny compared to everyday life where typical energies are many Joules, typical time are seconds

so the quantum effects are not normally apparent in everyday life because they are too small.



The deeper we go down the distance scale from

crystal
molecule
atom
atomic nucleus
protons & neutrons
quarks

the more apparent these quantum effects are

Subatomic physics

= nuclear physics
(study of the atomic nucleus)

+

particle physics
(study of the elementary particles that make up nuclei and atoms)

We shall apply these principles of quantum mechanics to several topics of subatomic physics

- the sizes and shapes of atomic nuclei
- the energy scale of nuclear processes
- the existence of quarks

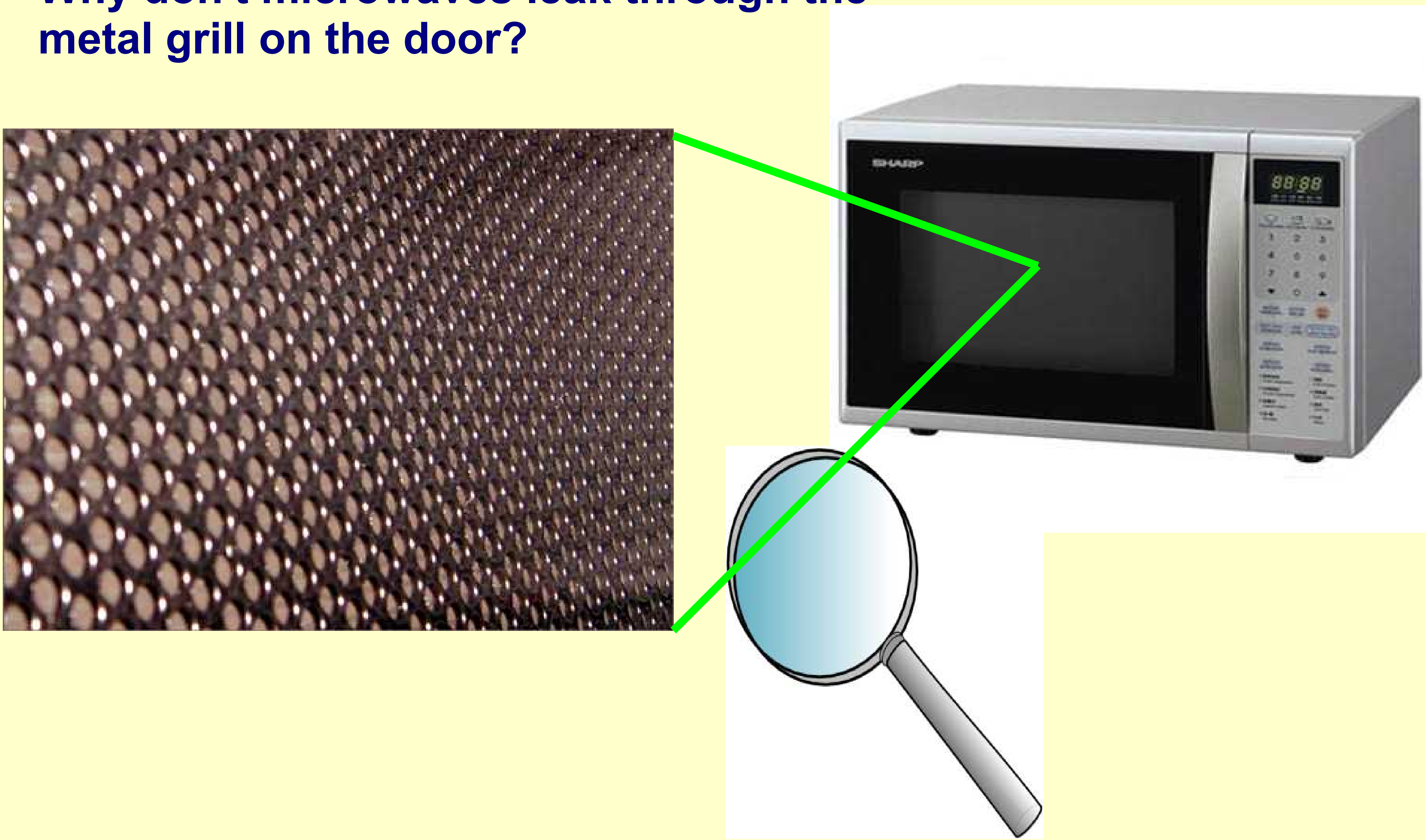
1. The sizes and shapes of atomic nuclei

How do we see atoms and subatomic structure?
Not with this! **WHY NOT?**

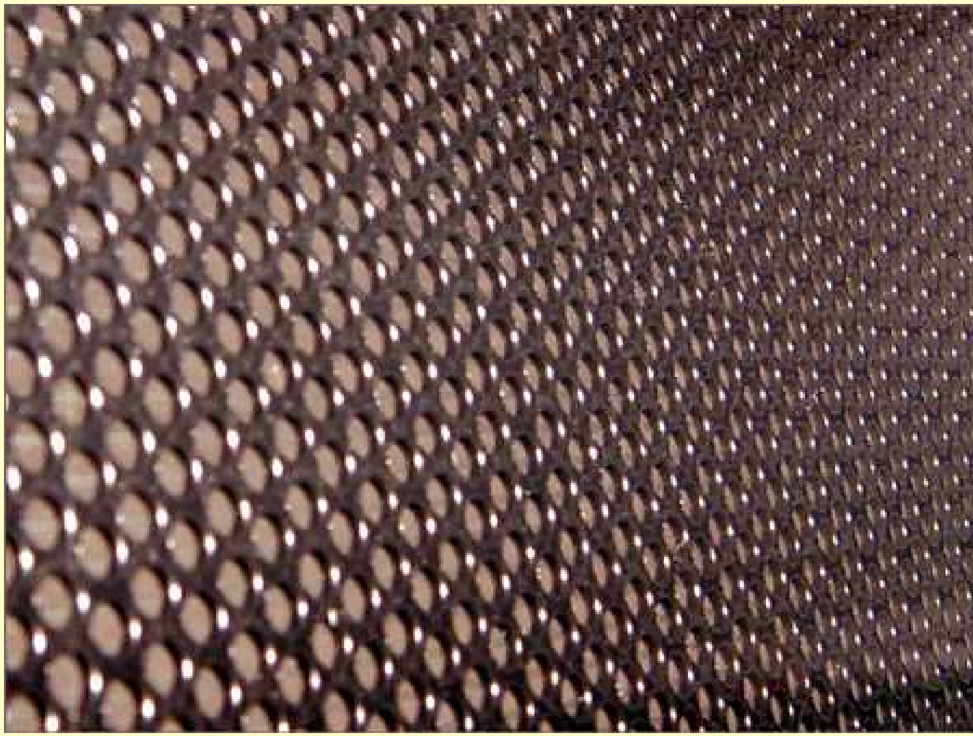
How do we see atoms and subatomic structure?
Not with this! WHY NOT?

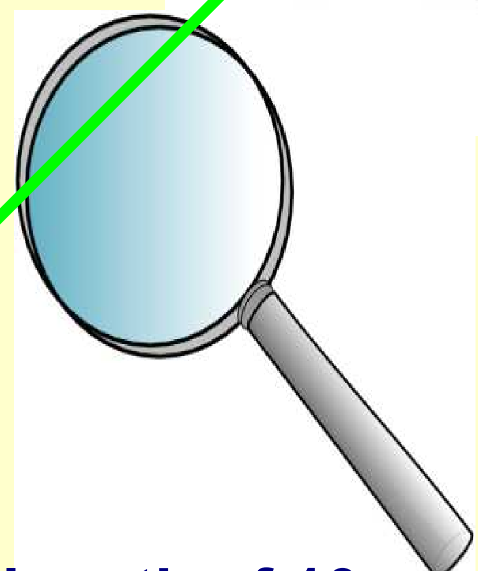
Because light is a wave, it has a “fuzziness” the size of its wavelength, and cannot resolve features smaller than that!

Why don't microwaves leak through the metal grill on the door?



Why don't microwaves leak through the metal grill on the door?

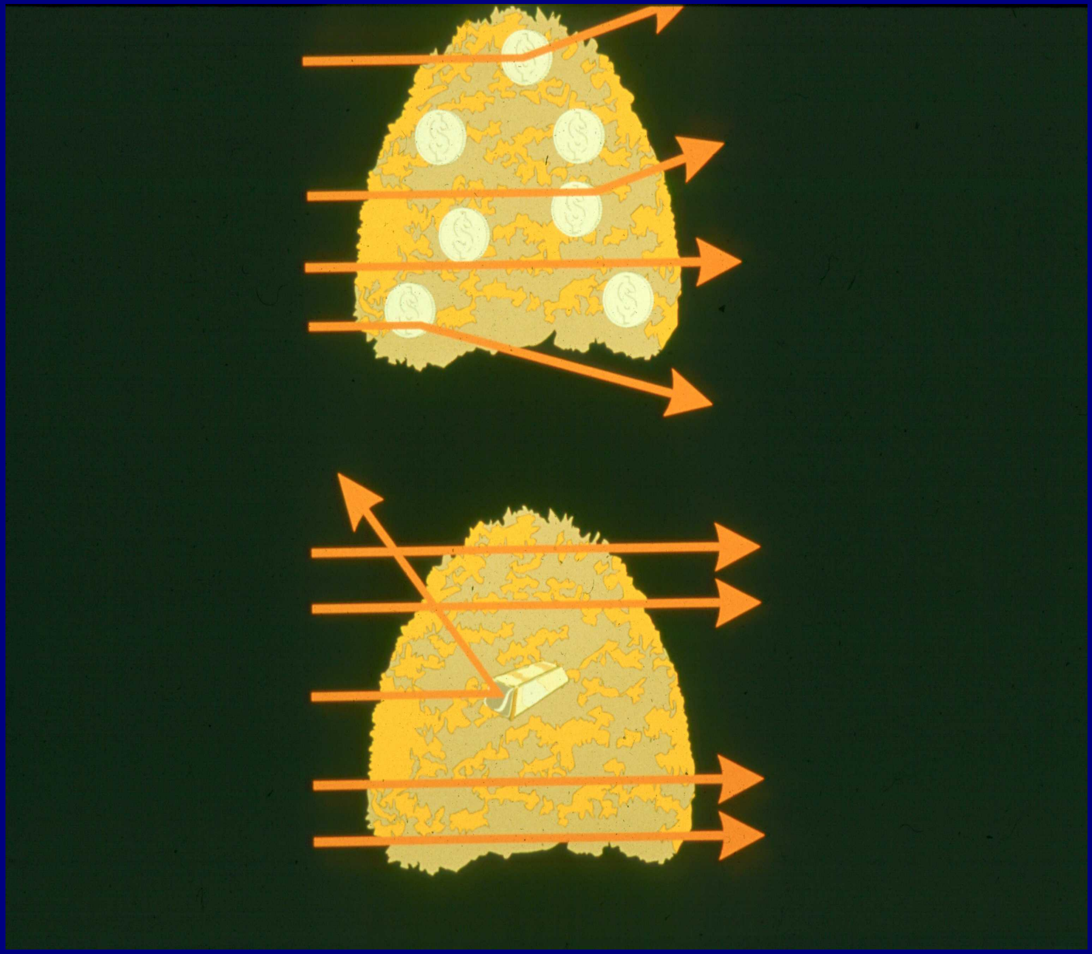




Because the waves have a wavelength of 10 cm, and cannot resolve the 2 mm holes in the grill. As far as they are concerned, the grill is solid metal!

How do we “see” what's inside an atom? By means of scattering experiments.

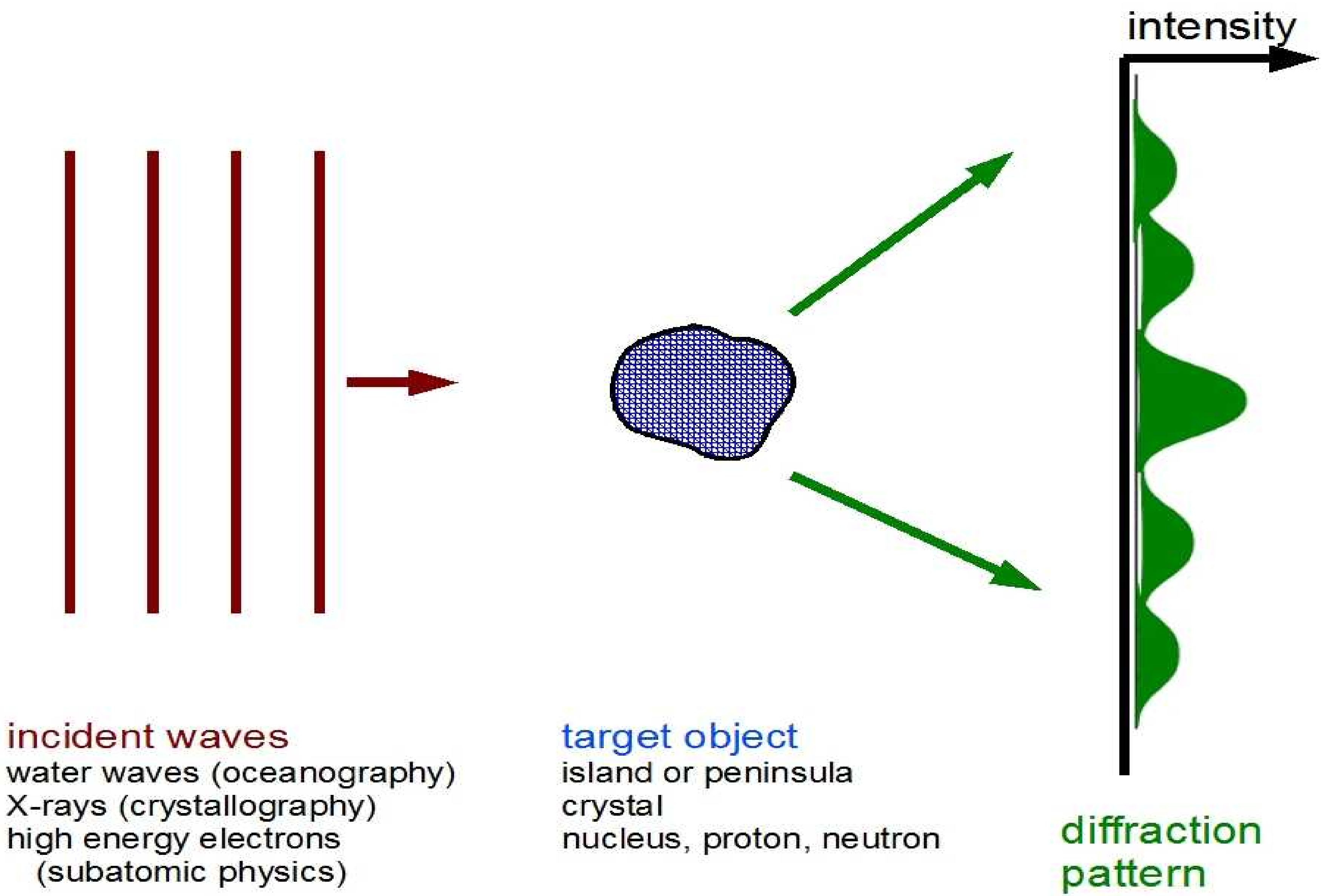
Gold coins in haystack analogy to how Rutherford discovered the atomic nucleus



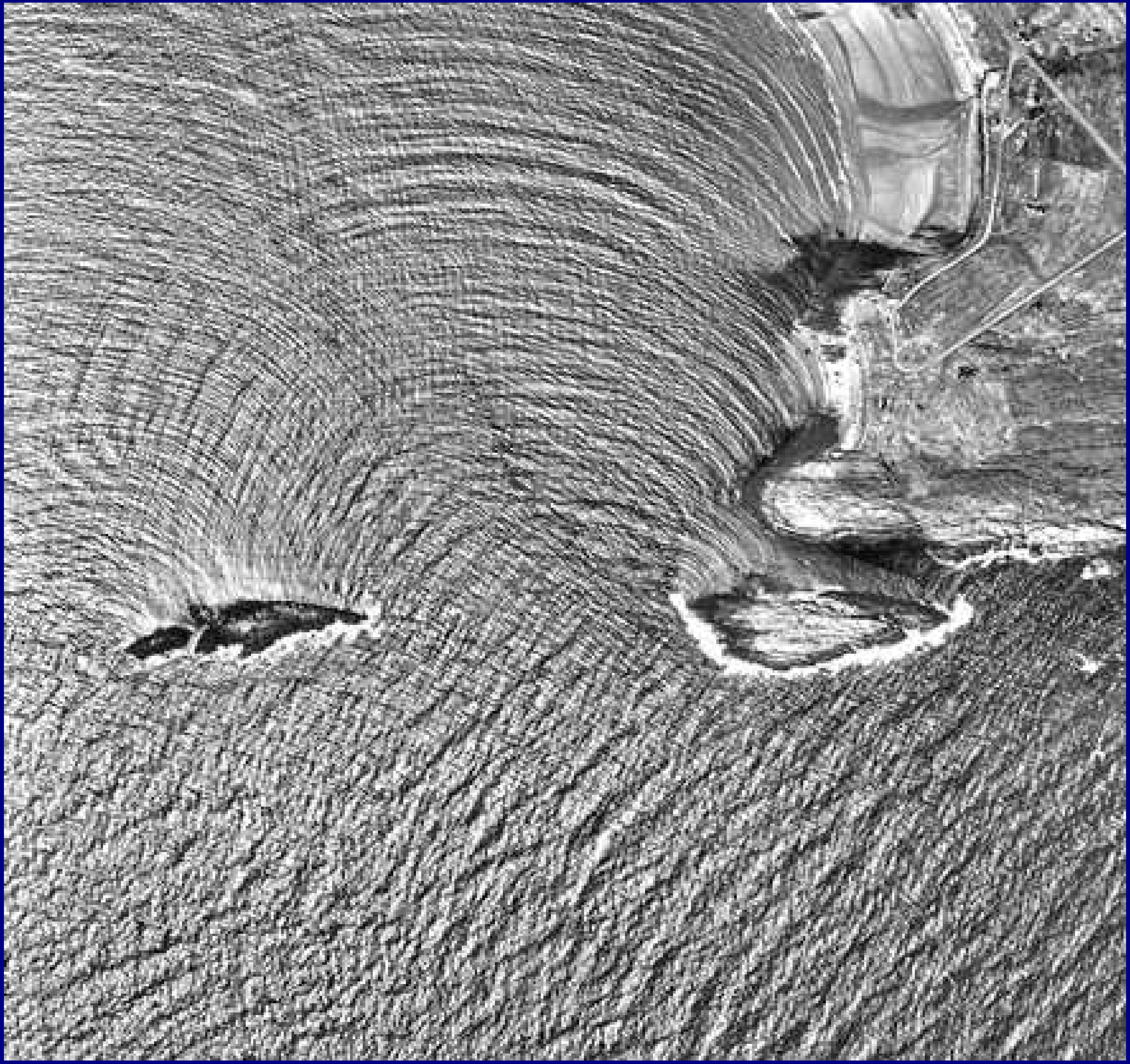
many small deflections of the projectiles

a few large deflections -- this is what the atom looks like!

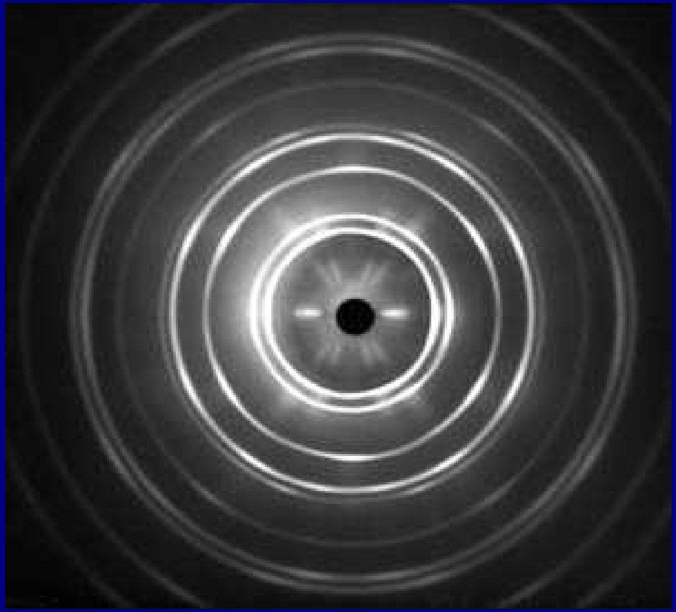
Bullets through haystack picture not accurate!
Actually, cannot neglect the wavelike nature of the projectiles!
In wave picture, a scattering experiment looks like this:

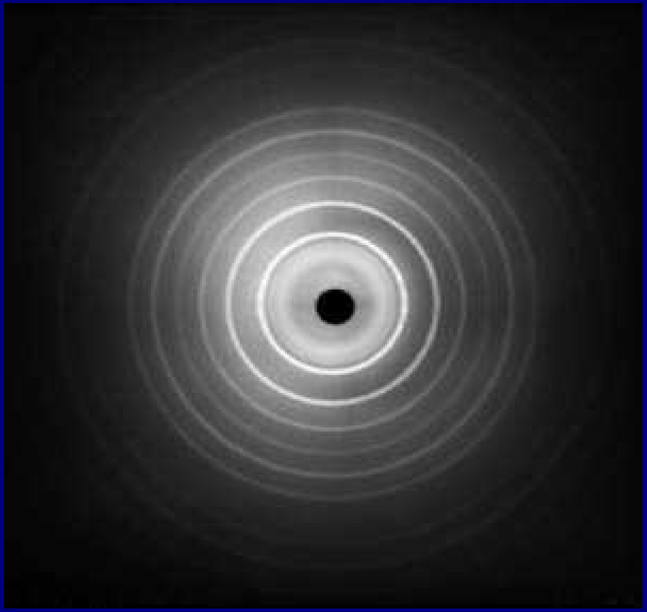


Diffraction of water waves



Diffraction of X-rays from powders of Al and NaCl
(powder = many tiny crystals in random orientations)

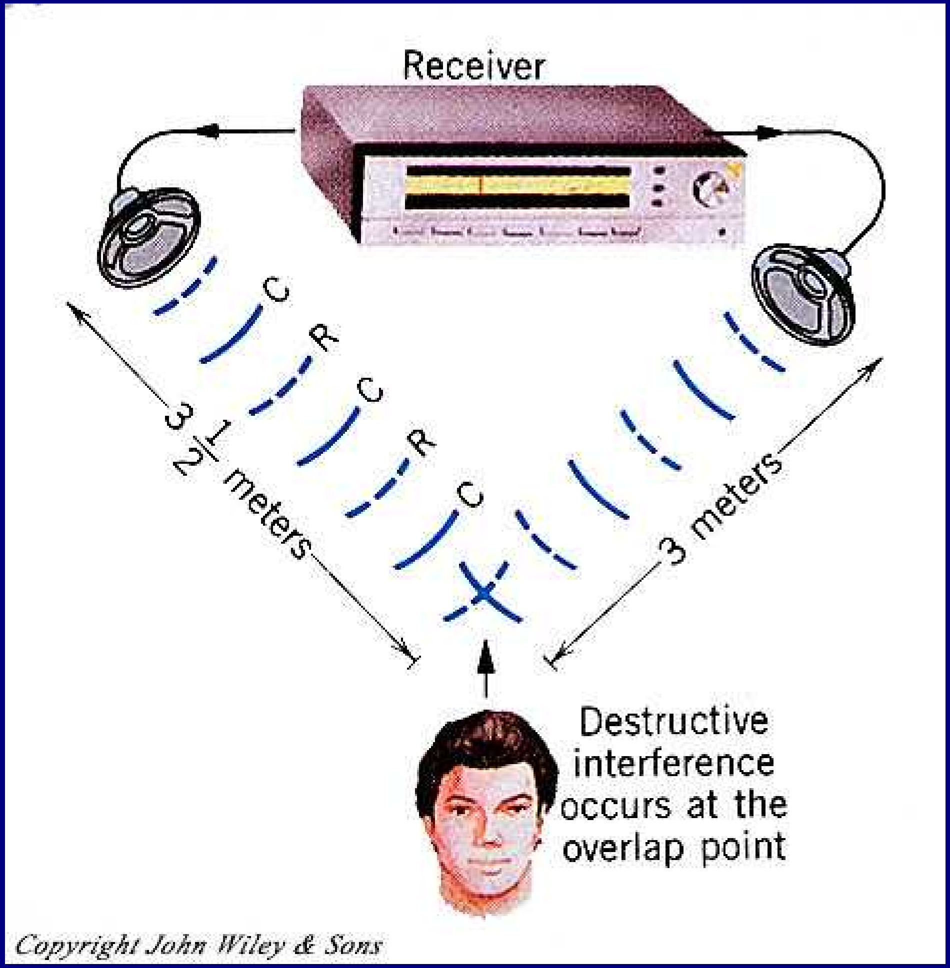




from C.C. Jones,   minerva.union.edu/jonesc/Scientific.html

Note that the intensity pattern for the two types of crystals are different.   By mathematically working backwards from the diffraction pattern, crystallographers can deduce the location of the atoms in the crystal.

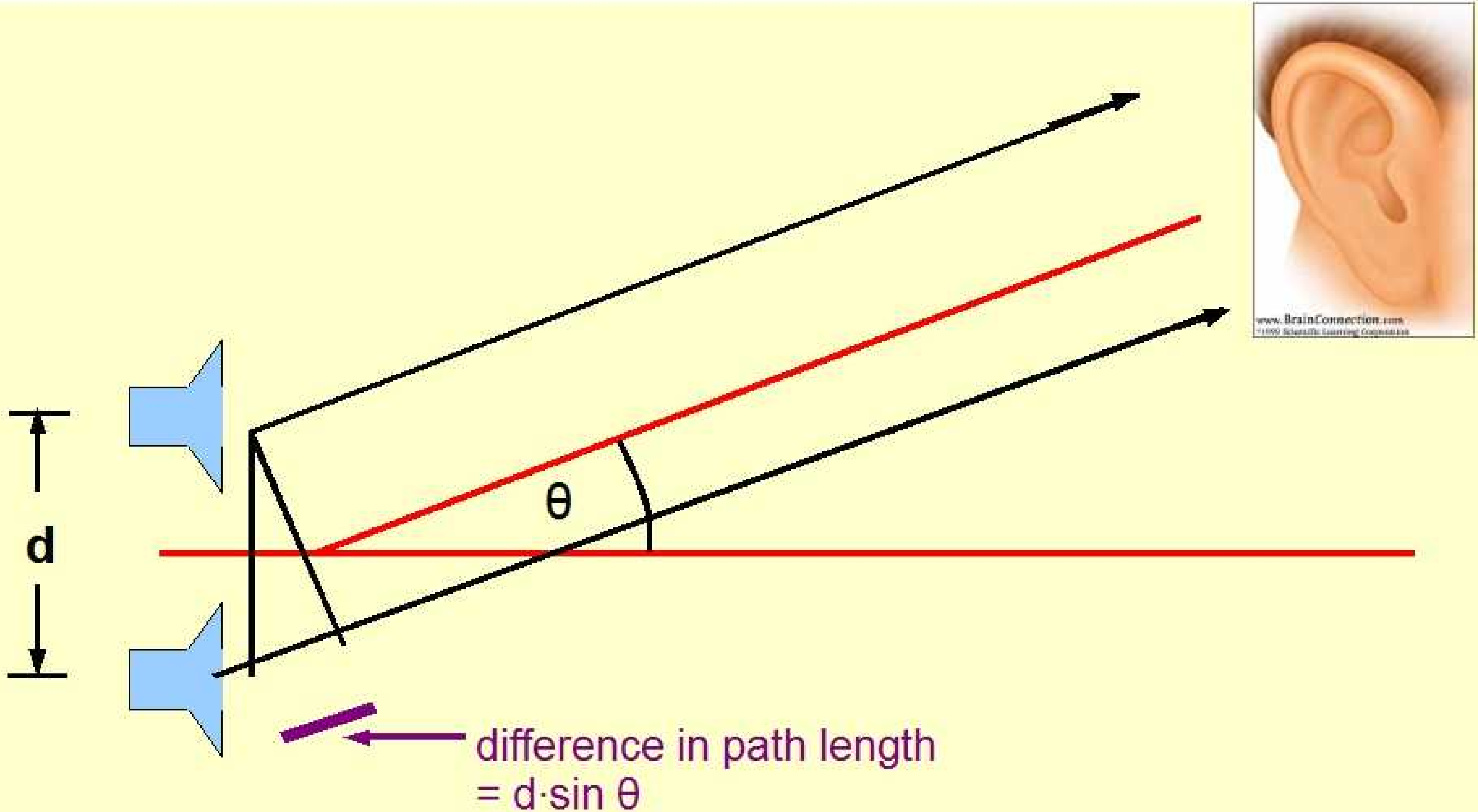
To get an idea of how the diffraction pattern can tell us the size and shape of the nucleus, consider the following demo:



If the distance from one speaker differs from the distance from the other speaker by exactly 1 wavelength or 2 wavelength, etc.

then the two crests arrive at the same time and there is constructive interference.

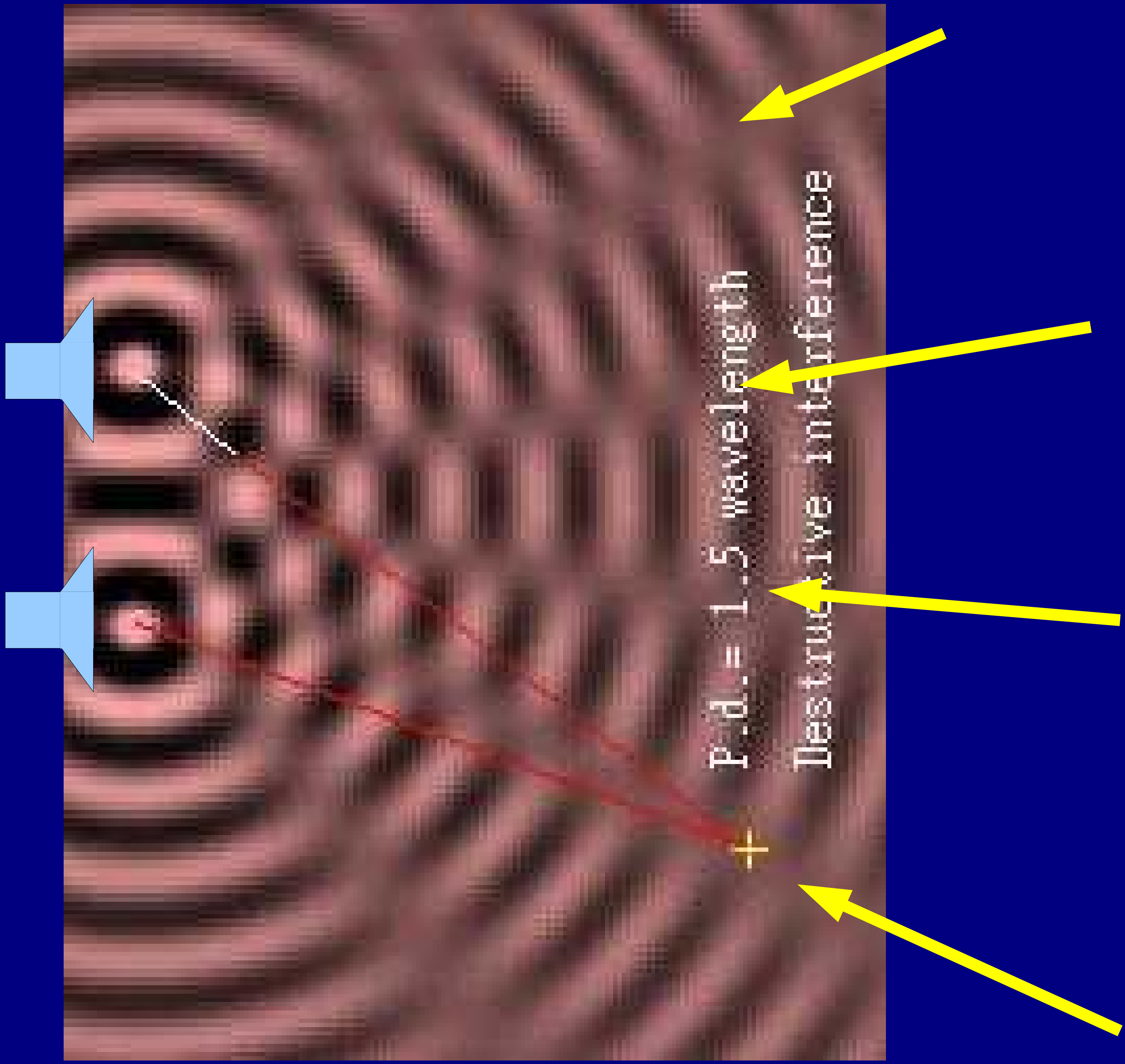
If the distances differ by a half-integer from these values, then there will be destructive interference.



For constructive interference $d \cdot \sin\theta = n\lambda$ or $\sin\Theta = n\lambda / d$

If d is small, separation between the adjacent bands of constructive interference is large.

Smaller separation between sources makes a wider diffraction pattern.



nodal
lines
of
complete
silence !

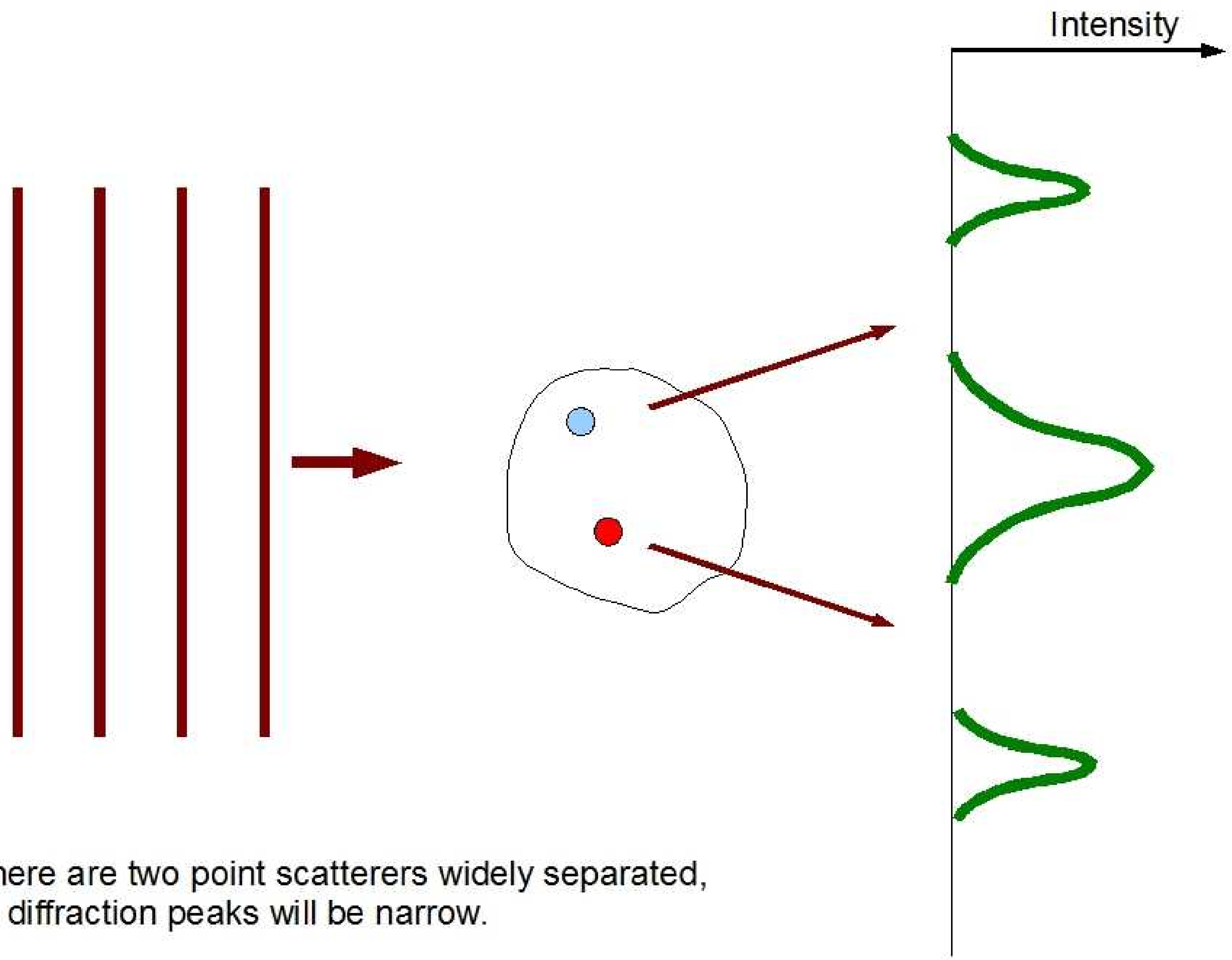
Let's try this demo with two loudspeakers!

Download a freeware sound generator, e.g. SoundGen, and set up 2 speakers about 100 cm apart, generate sound at 880 Hz frequency (λ=0.39 m) and move your head around to find the nodal lines at $\theta=23^0$ (n=1) and $\theta=51^0$ (n=2).

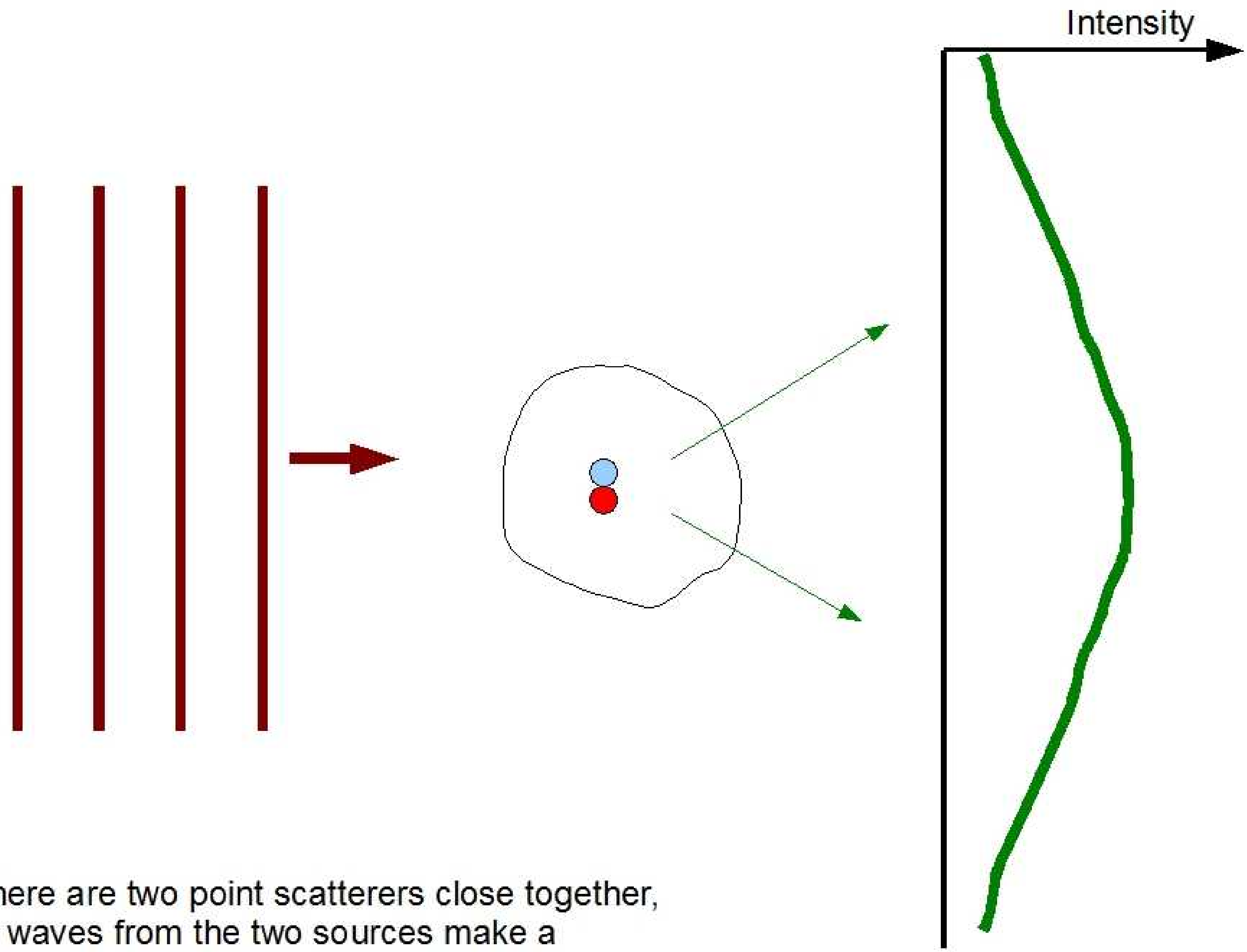
Reduce the separation between speakers to 50 cm and observe that the first nodal line moves out to $\theta=51^{0.}$

CONCLUSION:
Smaller (more point-like) source results in wider diffraction pattern.

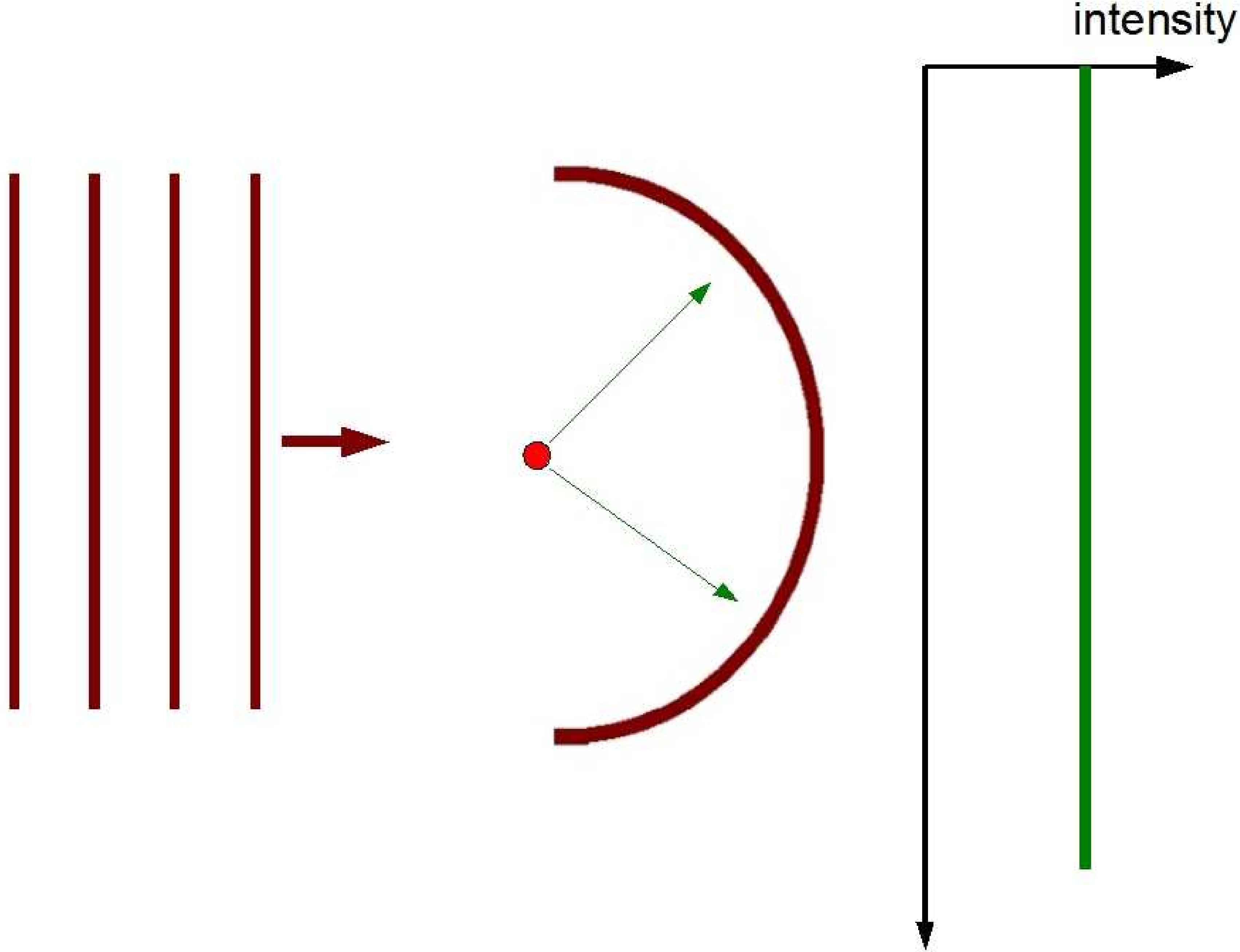
Q: What happens if we decrease the frequency?



If there are two point scatterers widely separated,
the diffraction peaks will be narrow.

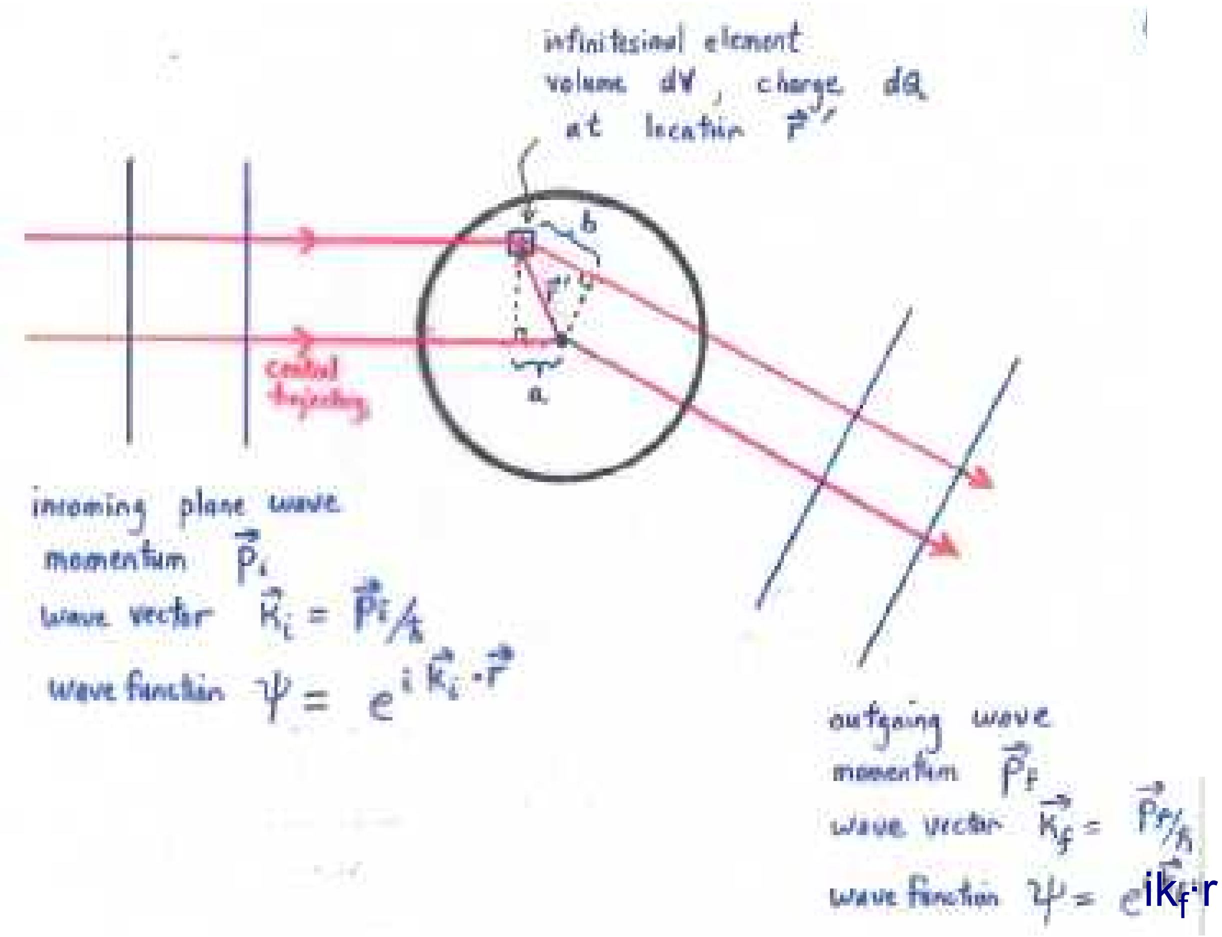


If there are two point scatterers close together, the waves from the two sources make a broad diffraction pattern



single point-like scattering centre
results in uniform scattering intensity

Suppose the scattering centers are smeared out and described by density distribution ρ(**r'**). In the case of charged particles interacting via the electric force ρ(**r'**) is the charge density distribution.



The total wave scattered in some direction is obtained by adding the waves from each of the infinitesimal elements, with the proper phase. After some simple math, we find

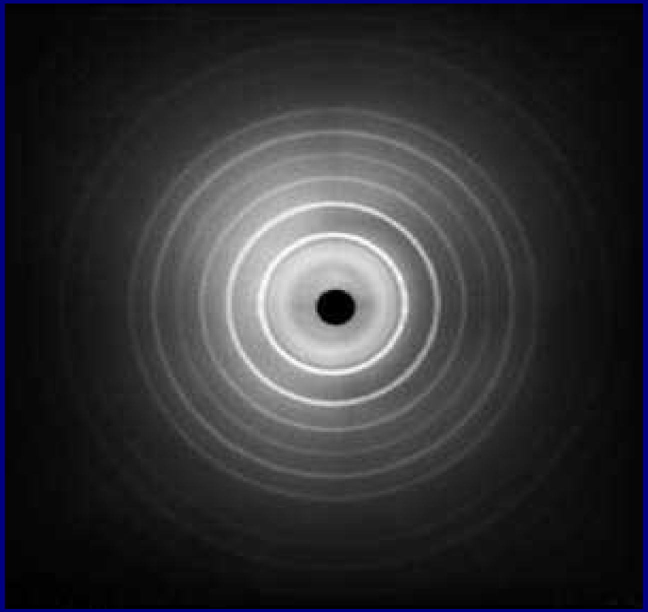
$$\text{Scattering Rate for extended Nucleus} = \text{Scattering Rate for point-like Nucleus} * \left| \underbrace{\int \rho(\mathbf{r'}) \exp(i\, \mathbf{q}\cdot\mathbf{r'})\, dV'}_{\text{Fourier transform of the charge density distribution } \rho(\mathbf{r'})} \right|^2$$

where $\mathbf{q} = (\mathbf{k_i} - \mathbf{k_f}) / \hbar$

For elastic scattering at angle θ $q = 2p / \hbar * \sin(\theta/2)$

By measuring the scattering rate at different angles θ, and sampling at different values of **q**, we can map out the square of the Fourier transform.

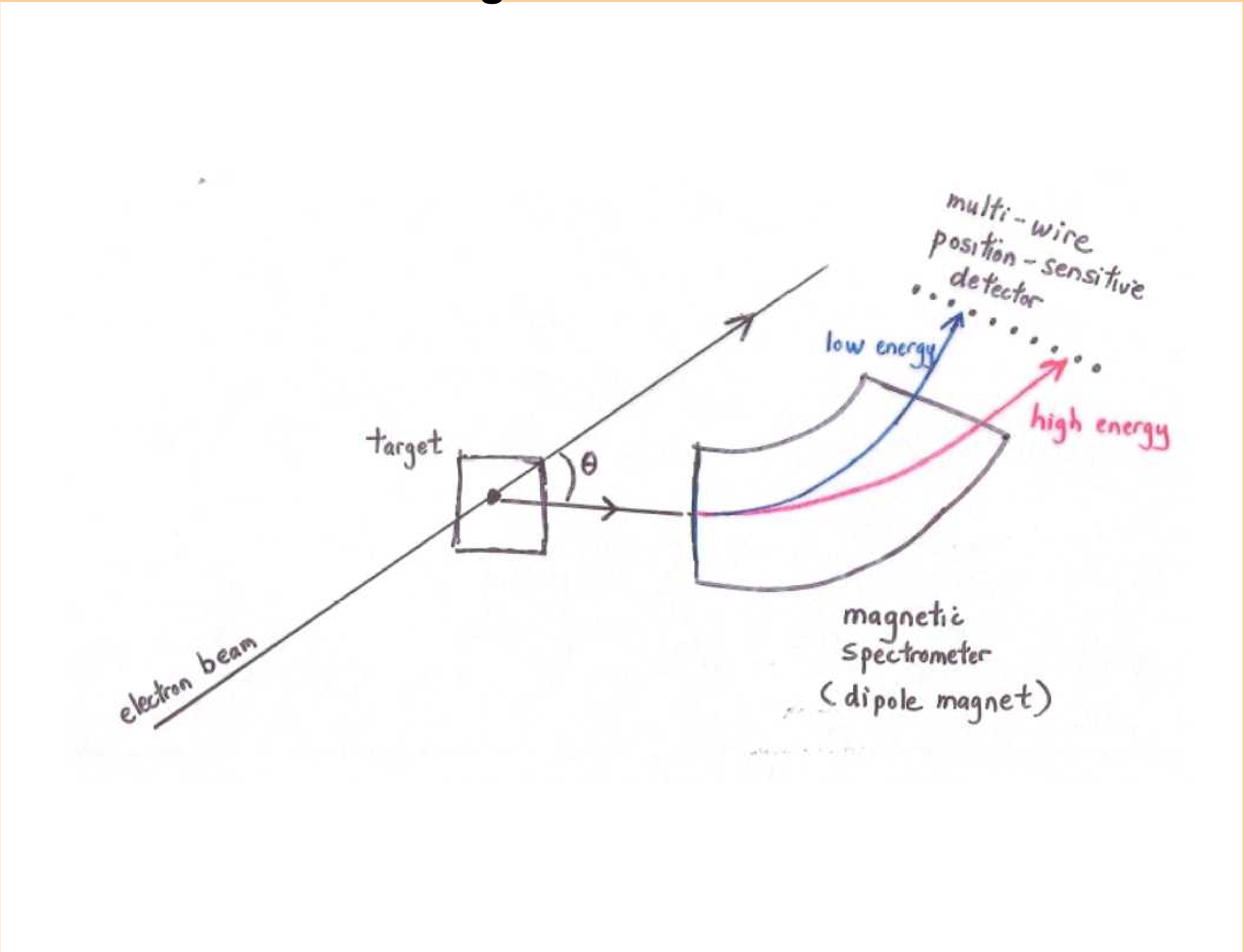
Then apply an inverse Fourier transform to get the nuclear charge density ρ(**r'**)



Thus, X-ray diffraction image in crystallography gives the squared Fourier transform of the electron density inside the crystal.

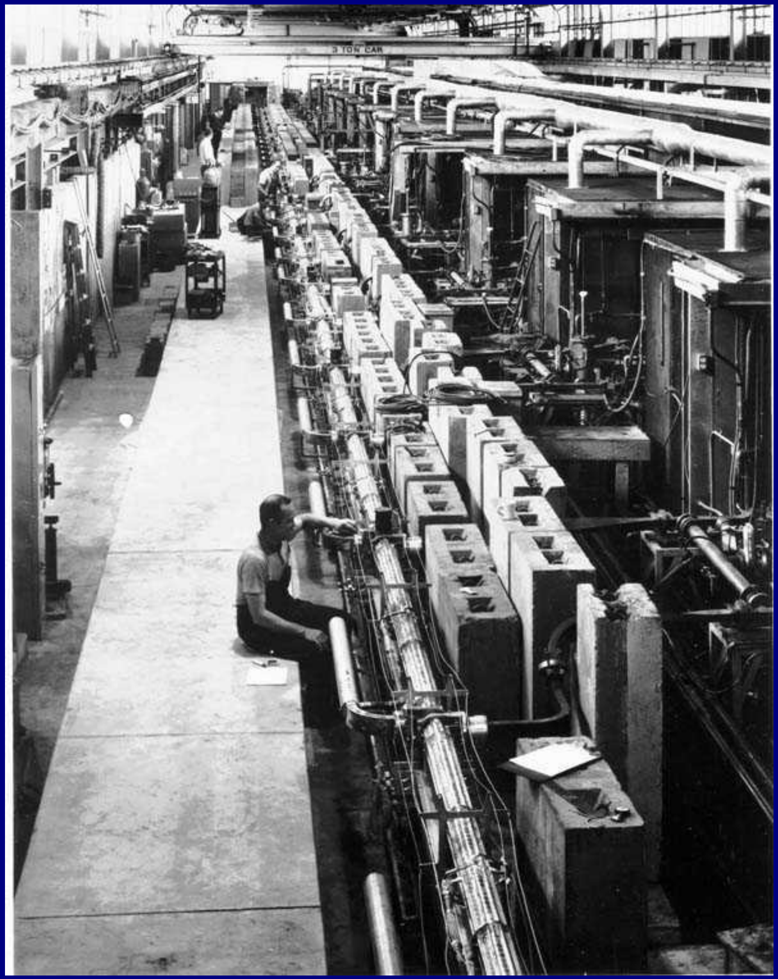
We can do the same thing, using electron waves to measure the charge density distribution inside the nucleus.

Now do the same thing with electron waves on a nuclear target.



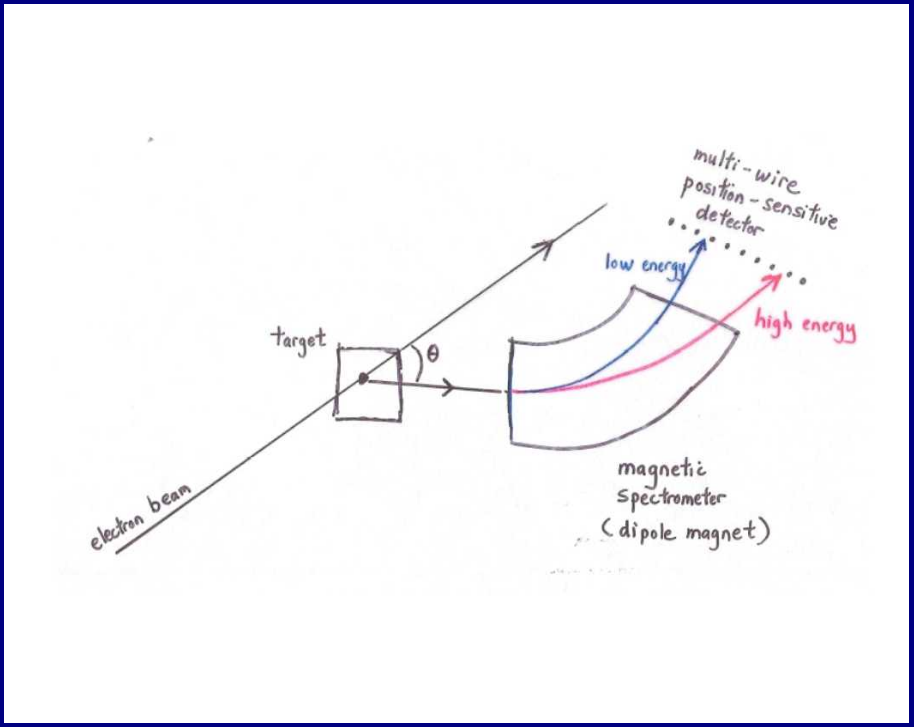
Measure the intensity of the scattered electrons at many values of the angle θ to map out the diffraction pattern. The width of the diffraction pattern will tell us the size and shape of the nucleus!

Robert Hofstadter
1961 Nobel Prize
in Physics for
work on elucidating
the charge distribution
of nuclei



Mark III electron linear accelerator
at Stanford (up to 1000 MeV energy)

Electron scattering from ^{12}C nucleus



4.43 MeV

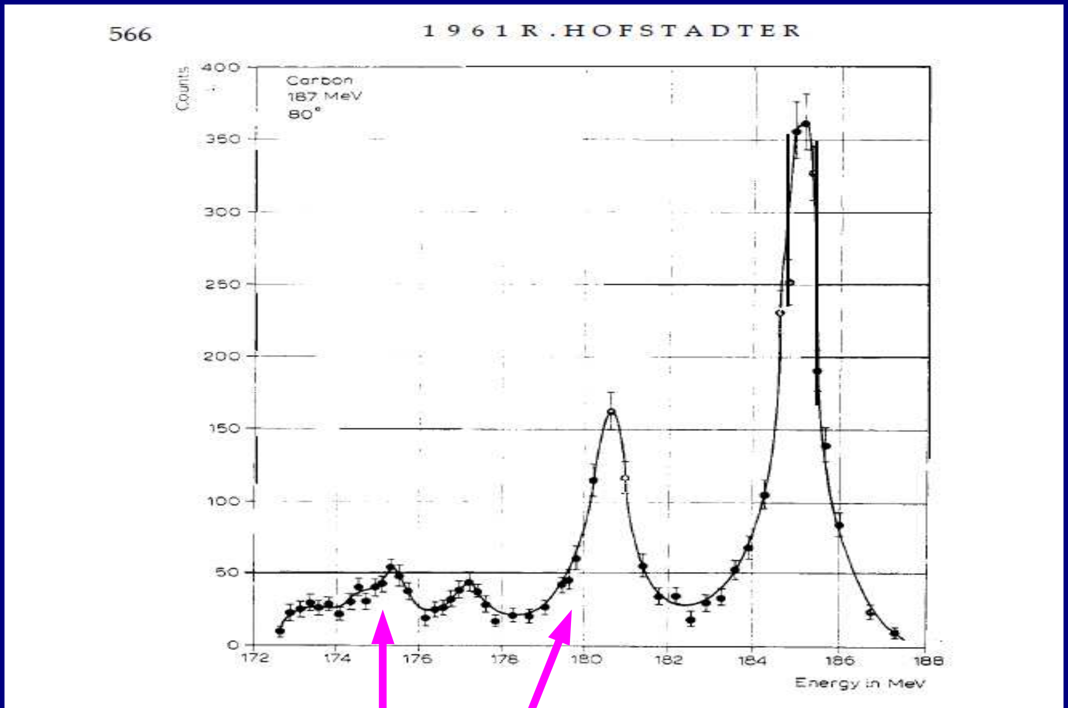


Fig. 4. This figure[8] shows the elastic-scattering peak from carbon at an abscissa near 185 MeV, and the inelastic-scattering peaks from the excited states of ^{12}C. The peak near 180.7 MeV is associated with the 4.43-MeV level.

inelastic scattering
peaks, where
nucleus is left
in an excited state
(vibrating or
rotating)

elastic scattering peak:
target nucleus is left
in its initial state

from Krane, Introductory Nuclear Physics

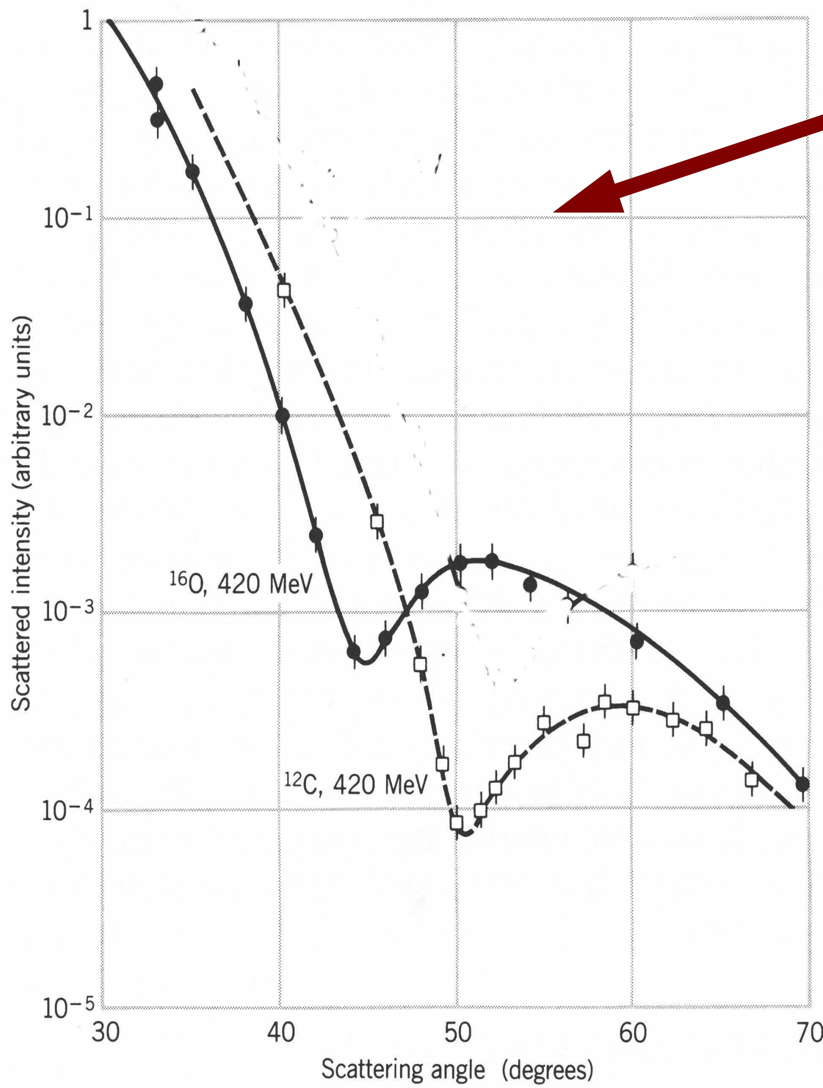
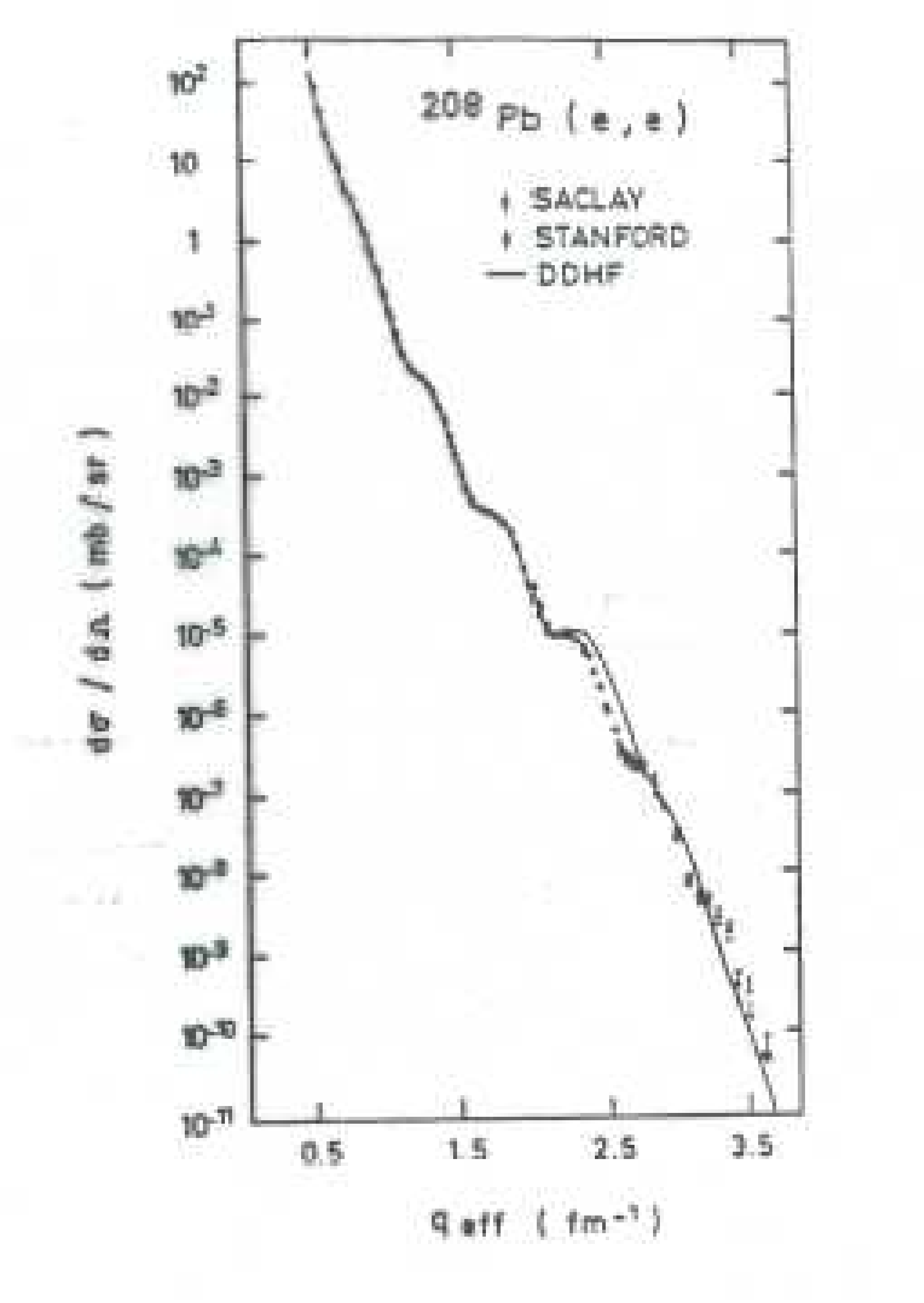


Figure 3.1 Electron scattering from ^{16}O and ^{12}C. The shape of the cross section is somewhat similar to that of diffraction patterns obtained with light waves. The data come from early experiments at the Stanford Linear Accelerator Center (H. F. Ehrenberg et al., *Phys. Rev.* **113**, 666 (1959)).

diffraction pattern of electron waves elastically scattering from nuclei of carbon-12 and oxygen-16.

Note that carbon-12 is smaller, so it produces a wider diffraction pattern.



Data for elastic scattering of electrons from ^{208}Pb. Note that the cross section spans 11 orders of magnitude. This means that the data at the right hand side come in at a rate 10^{11} more slowly than the data at the left hand side -- a real experimental challenge to measure these low cross sections.

By measuring the cross section, extracting the form factor squared, and doing the inverse Fourier transform, the charge distribution of the nucleus is obtained.

From the diffraction pattern, we can mathematically work backwards using an inverse Fourier transform to get the density profile of the electric charge in the atomic nucleus:

Notice how small the nucleus is:

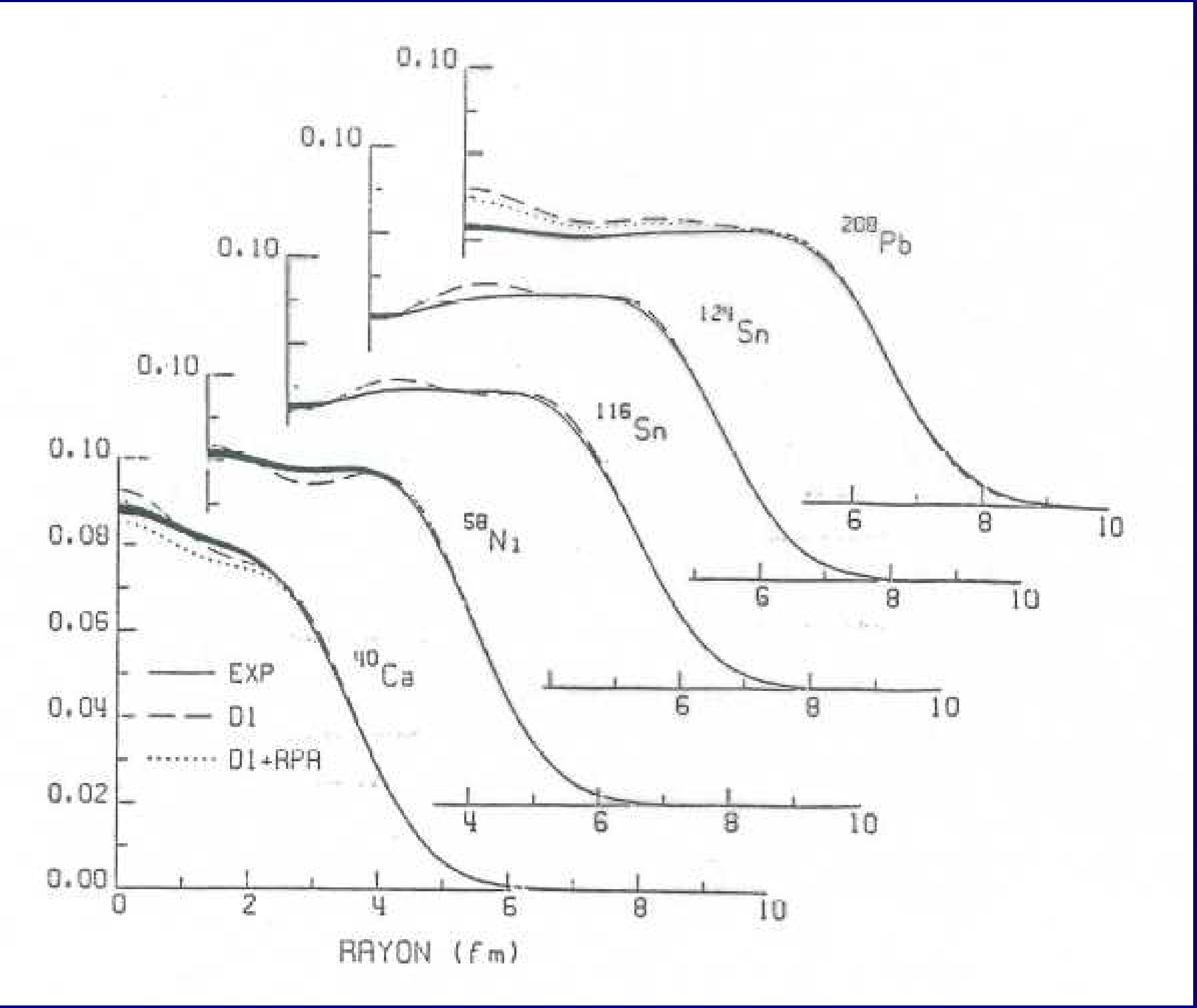
A Calcium nucleus has a radius of about 4 femto-metres) i.e.

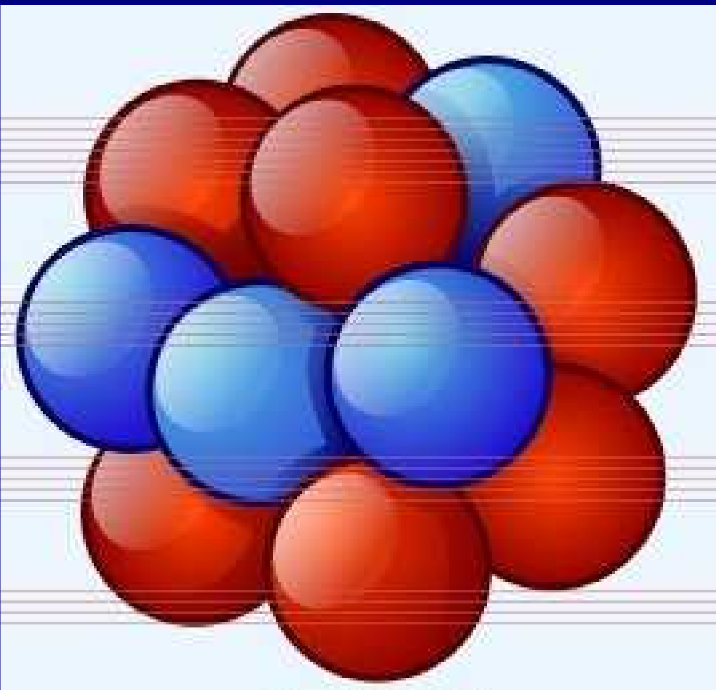
4×10^{-15} metre

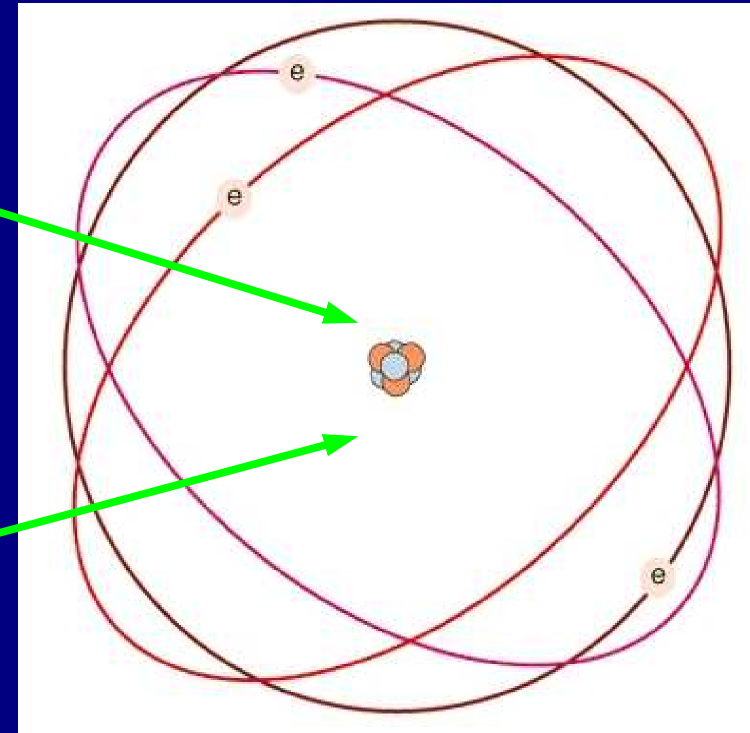
while the radius of a calcium atom has a radius of

180,000 femtometres

i.e. the nucleus is 45,000 times smaller in diameter !







Nucleus in an atom is like a pea in a football stadium.

Even though the atomic nucleus is so tiny compared to the entire atom, it contains 99.97% of the mass of the atom.

The nuclear matter is extremely dense – a teaspoon full would have a mass of 460 million metric tons!

Where in the universe can we find bulk quantities of nuclear-density matter?

Even though the atomic nucleus is so tiny compared to the entire atom, it contains 99.97% of the mass of the atom.

The nuclear matter is extremely dense – a teaspoon full would have a mass of 460 million metric tons!

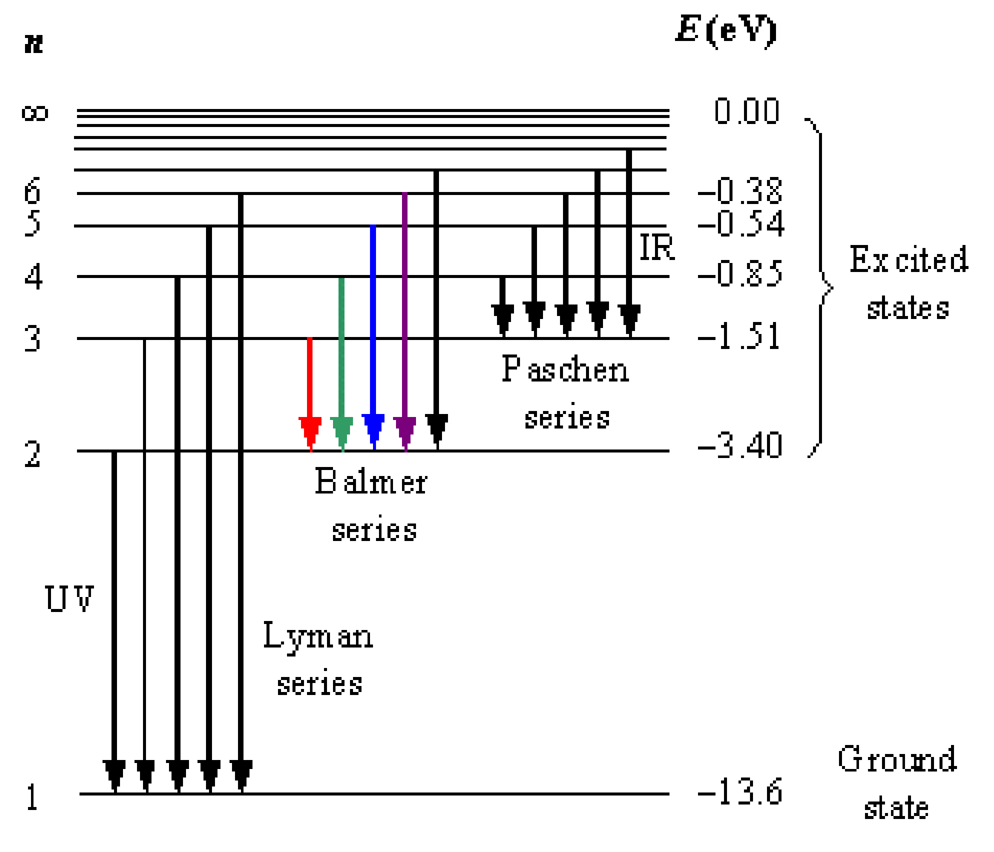
Where in the universe can we find bulk quantities of nuclear-density matter?

Answer: In neutron stars.

A neutron star is formed at the end of the life of a massive star. The pull of gravity is so strong that the protons in the atomic nuclei absorb the orbital electrons, forming neutrons. This allows the whole system to become more compact, thereby lowering the gravitational potential energy. The neutron star is basically a giant nucleus, about 25 km in diameter, but having a mass of between 1.4 and 2.1 solar masses, composed almost exclusively of neutrons.

2. The nuclear energy scale

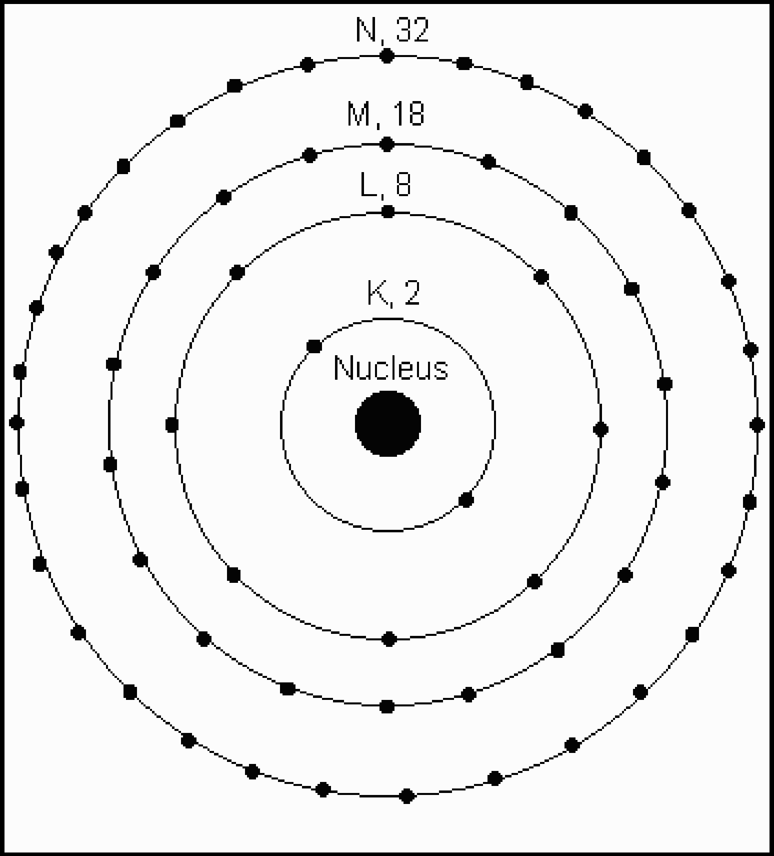
Atoms in excited states can de-excite and give off light, of energies typically around 1 electron-volt.

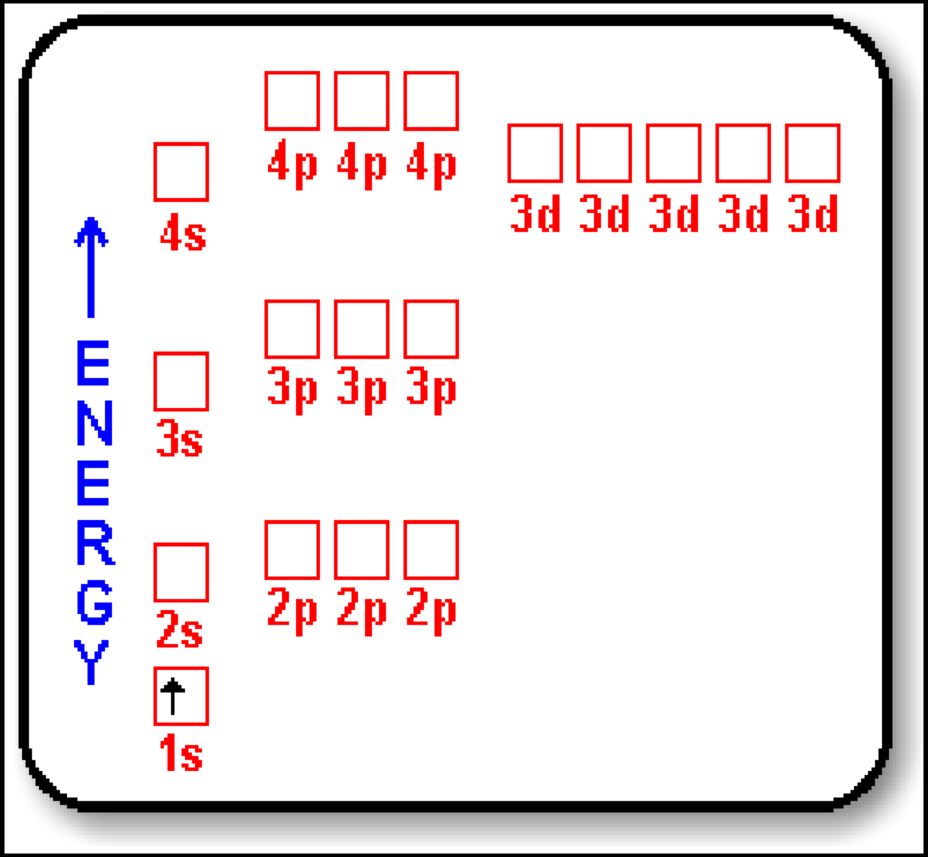


Energy levels of the hydrogen atom with some of the transitions between them that give rise to the spectral lines indicated

http://hyperphysics.phy-astr.gsu.edu /hbase/phyopt/grating.html

The electrons in atoms (and molecules) are arranged in shells.
Excited atoms and molecules are made by boosting the electrons to higher shells.





Similarly, the protons and neutrons in atomic nuclei are arranged in shells, and excited nuclei are made by boosting the protons and neutrons to higher shells.

Why are nuclear energies so high?

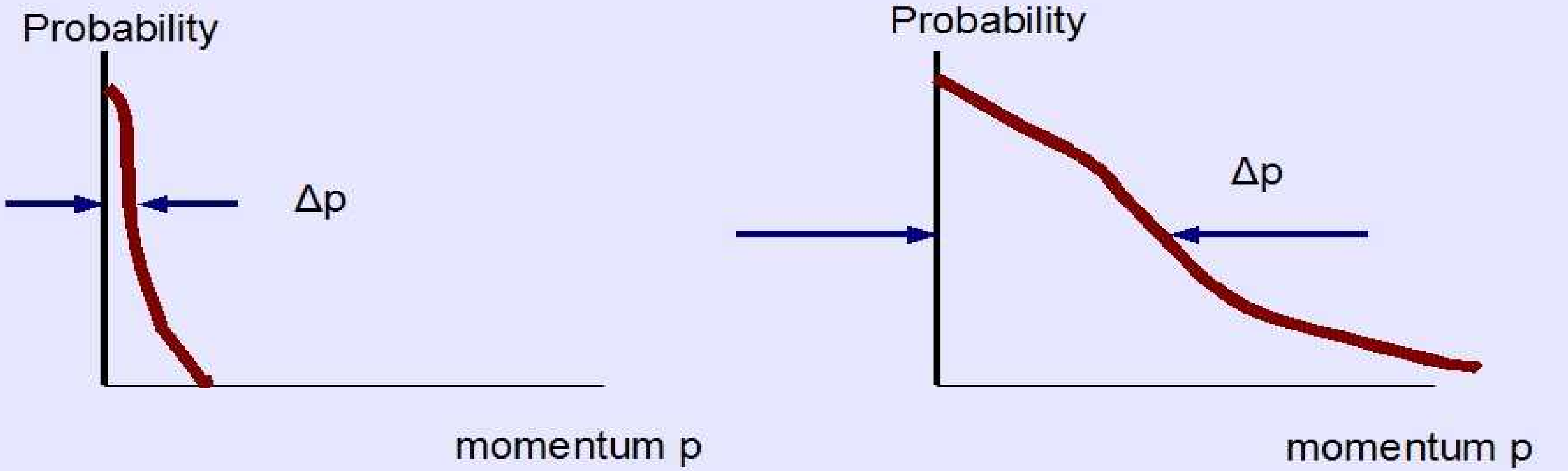
Because the particles are packed into a very small volume

Using the Heisenberg uncertainty principle

$$\Delta p \cdot \Delta x \geq h/4\pi$$

we can calculate the extent Δp that the momentum is smeared out.

Since the nucleus is 45,000 smaller, the momentum of the protons and neutrons is smeared out 45,000 times more than the electrons in an atom. Roughly speaking, the protons and neutrons have 45,000 more momentum



Atom:
Large confinement space Δx
Small momentum range Δp

Nucleus:
Small confinement space Δx
Large momentum range Δp

Recall that kinetic energy $KE = \frac{1}{2} mv^2 = p^2 / 2m$

and that protons are 1836 times more massive than electrons. So we get

$p_{nuclear} / p_{atomic} = 45{,}000$

$KE_{nuclear} / KE_{atomic} = (45{,}000)^2 / 1836 = 1.1$ million

The kinetic energy of protons in the nucleus is about 1 million times larger than the kinetic energy of electrons in an atom, just by the Heisenberg uncertainty principle, and in good agreement with experimental data.

The high energy of nuclear processes is an inevitable consequence of the small size of the nucleus + quantum mechanics.

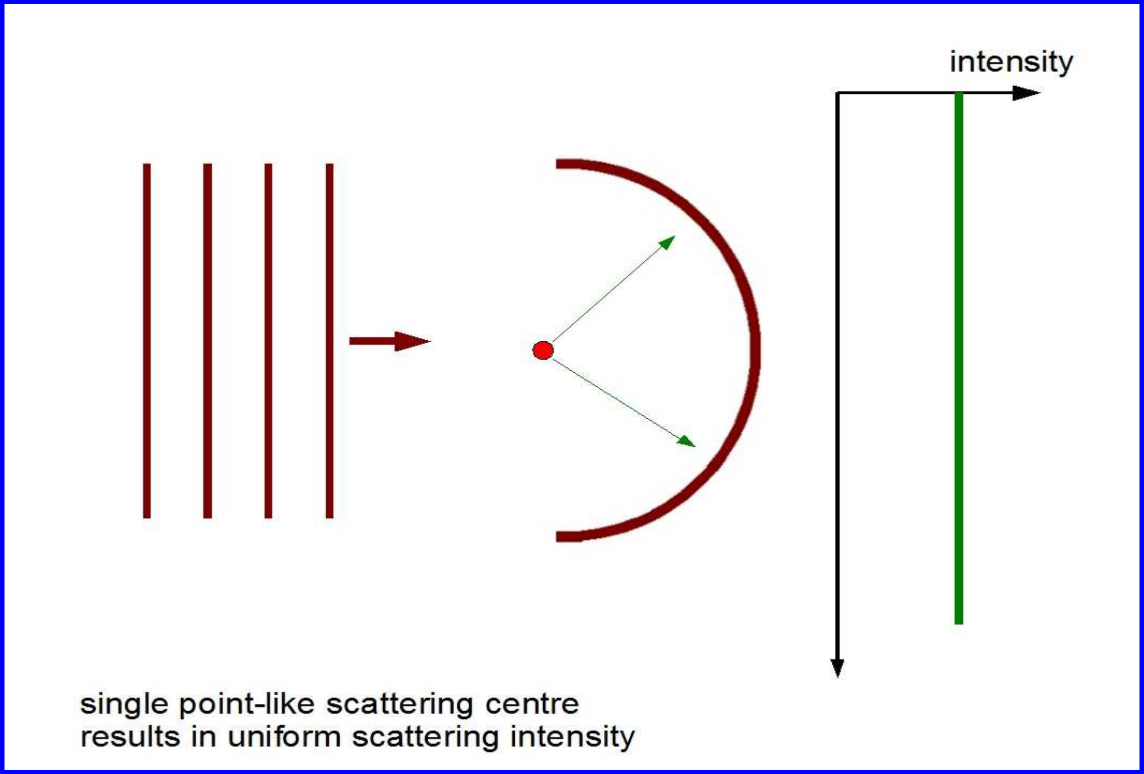
This is not so in classical mechanics. The kinetic energy of the tiny gears in a watch is not much higher than the kinetic energy of the flywheel in a turbine of a hydroelectric generating station.

3. The discovery of quarks

By the 1940's it was known that atomic nuclei were made of protons and neutrons. But do protons and neutrons have any size? Do they have smaller constituents inside of them?

Put a liquid hydrogen target into the electron beam and see what the diffraction pattern looks like!

Recall for a point-like proton with no size, we expect this:

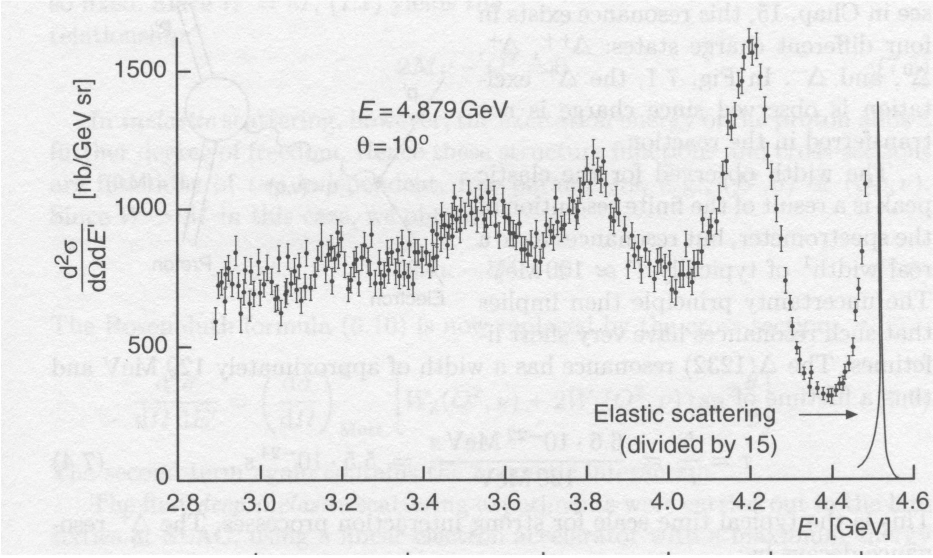


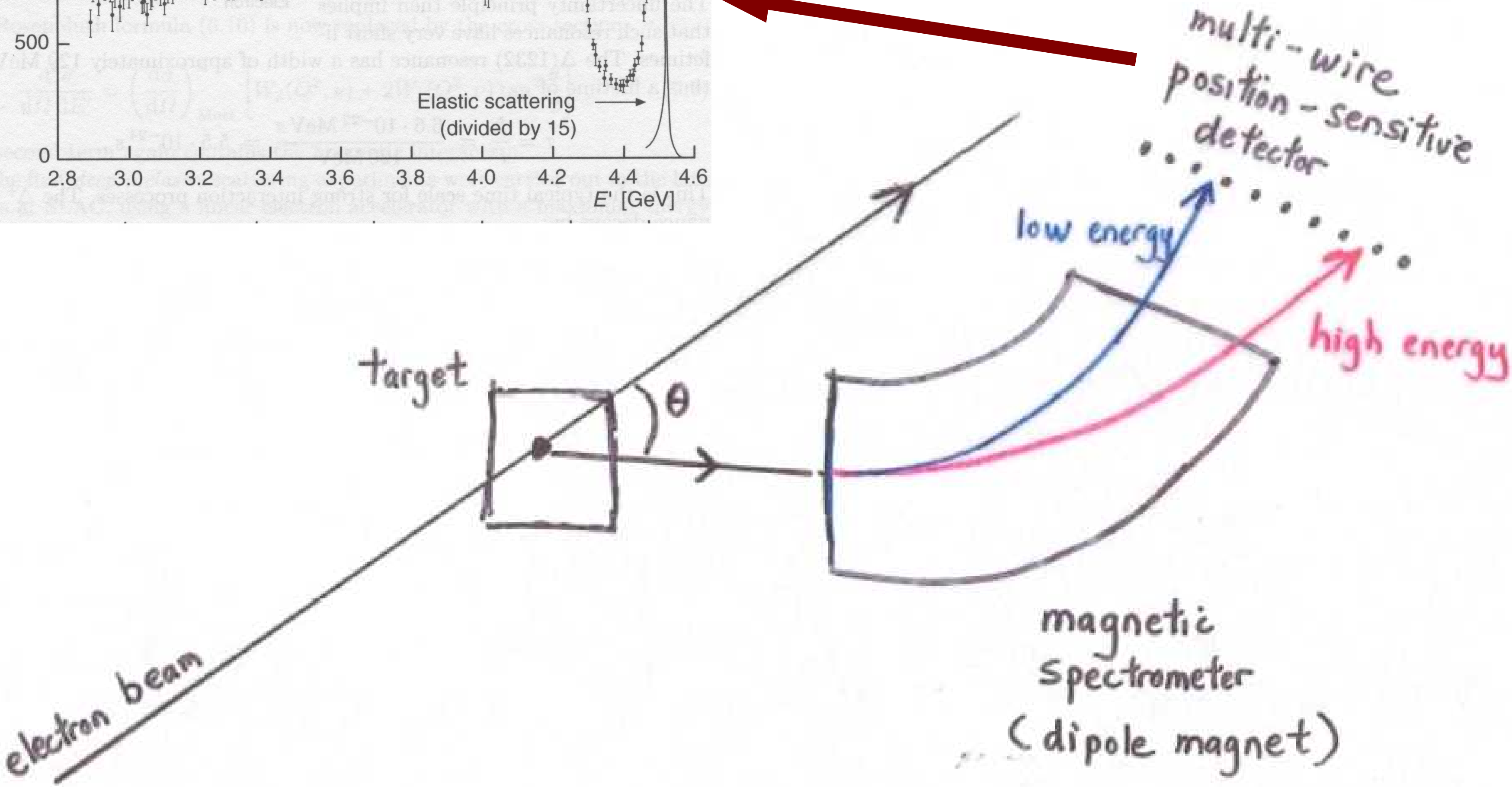
Stanford Linear Accelerator Centre – a 2-mile long electron accelerator, which boosts electrons to an energy of 20 billion electron volts (later upgraded to 60).

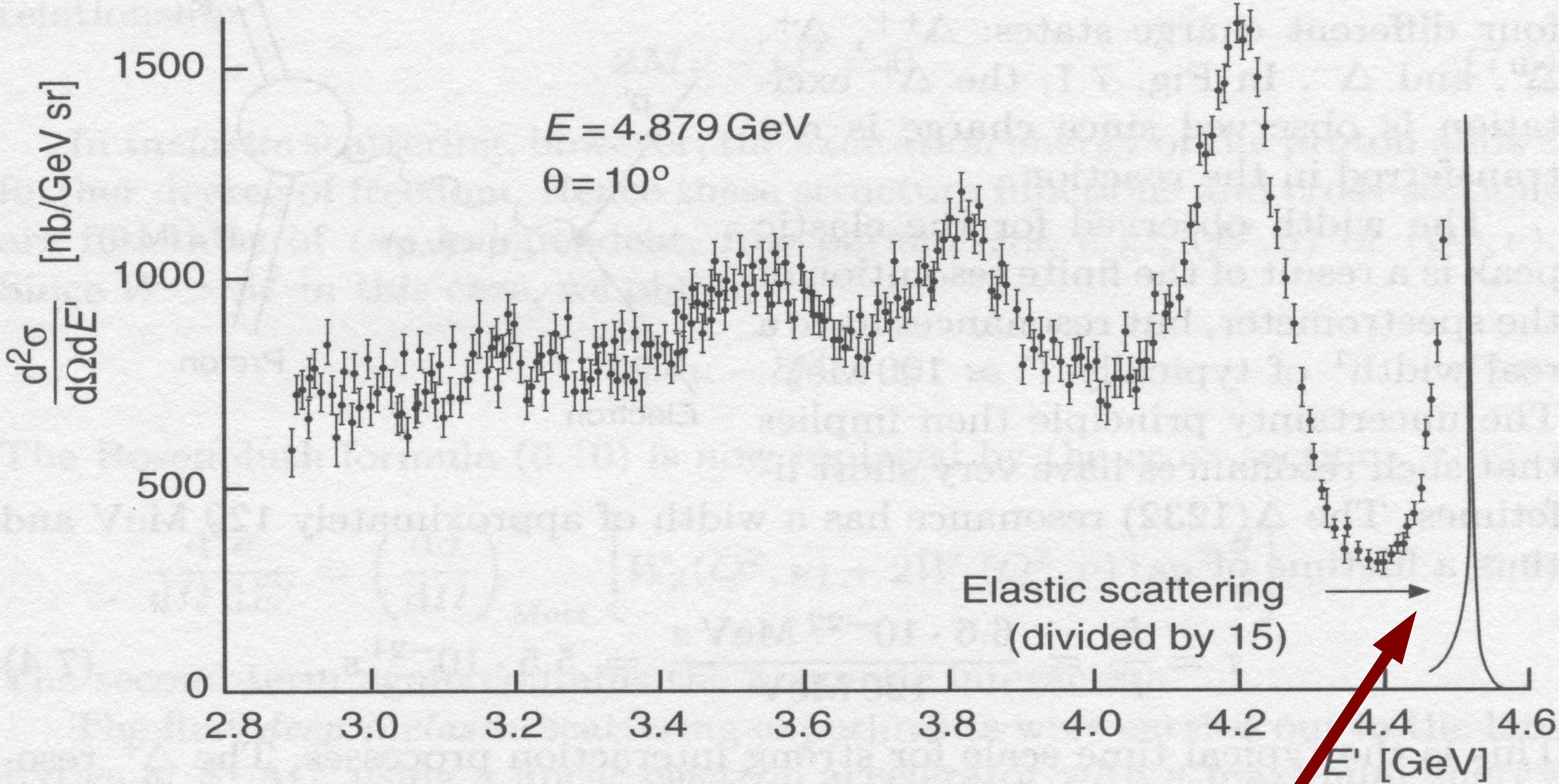
The de Broglie wavelength of these electrons is about 1/100 fm, which is about 1% the radius of a proton. So this machine can resolve features deep inside the proton.

A view of the gigantic magnetic spectrometer used to analyze the energy of the scattered electrons. Note the two people near the bottom for scale.

each of the peaks corresponds to different scattering processes

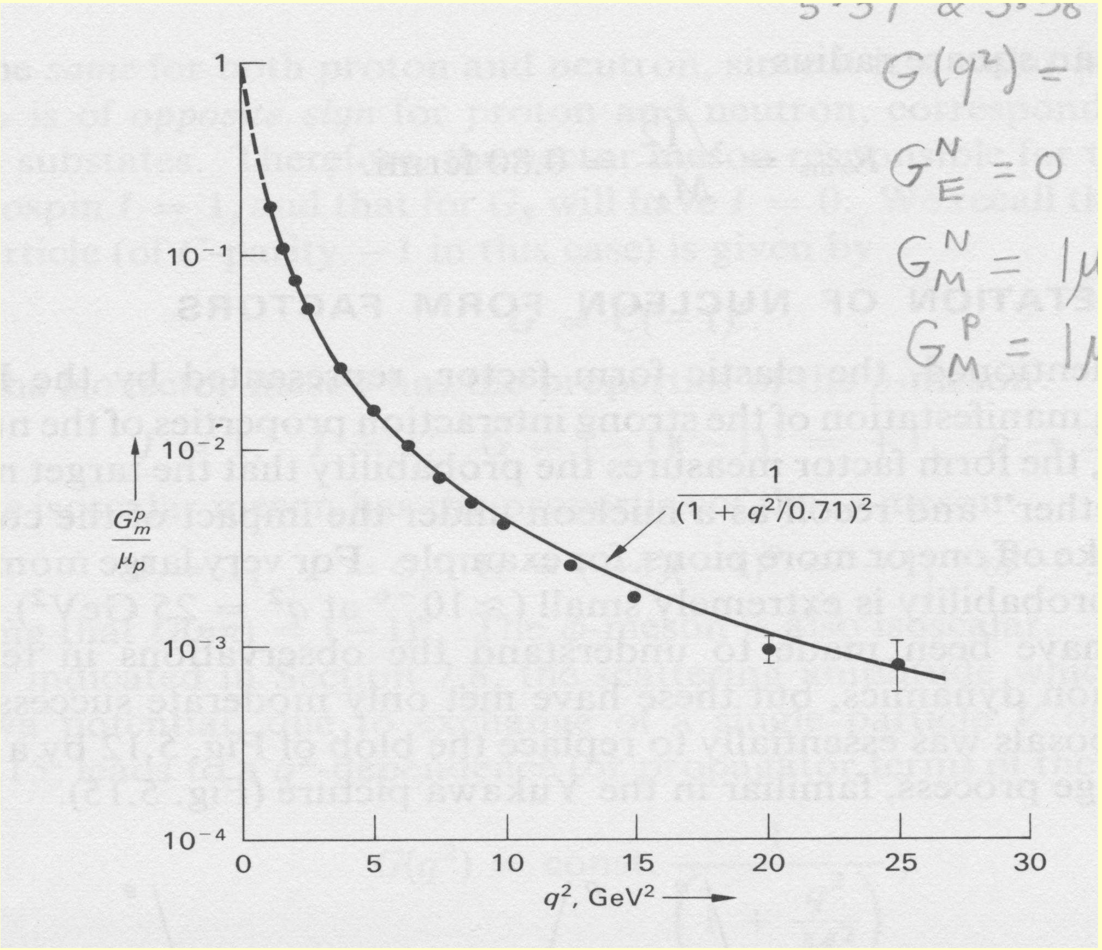


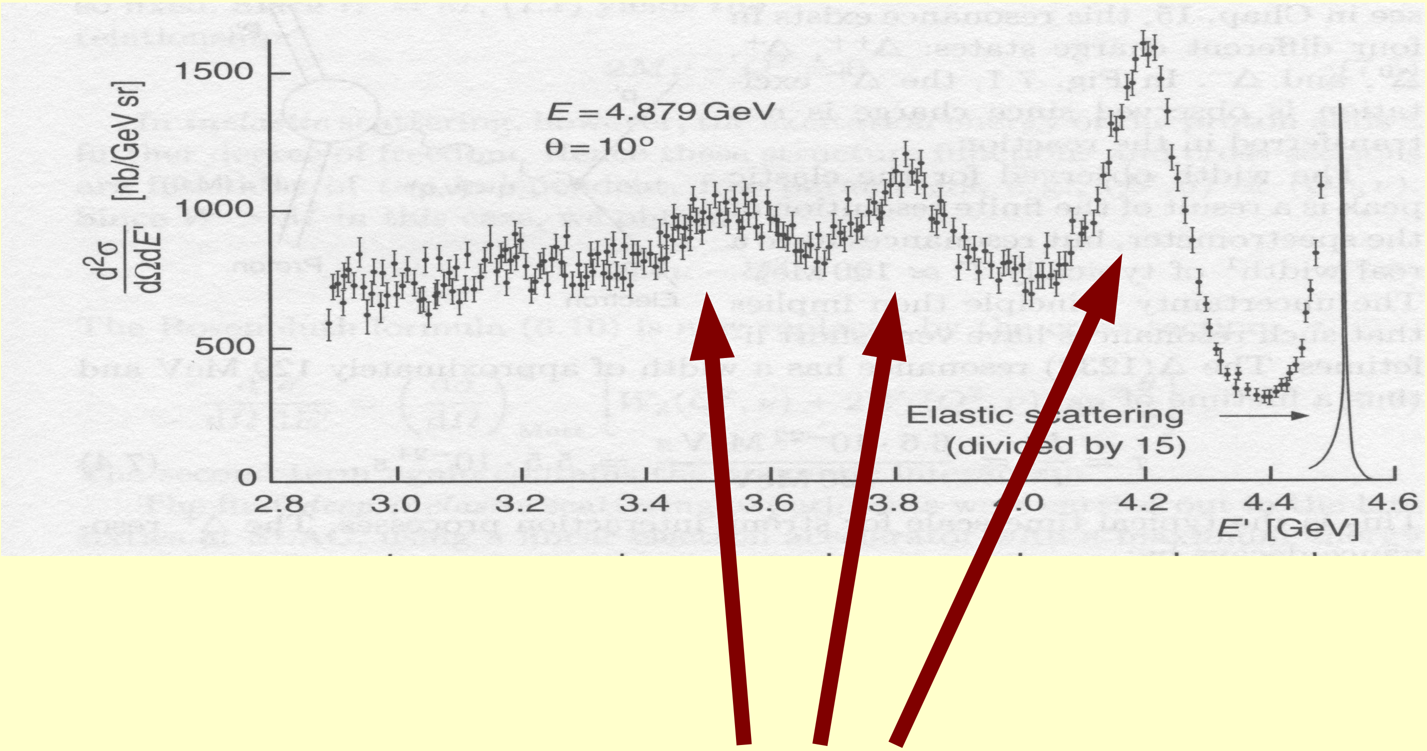




highest energy scattered electrons correspond to elastic scattering, where the electron just bounces off the proton and leaves the proton intact

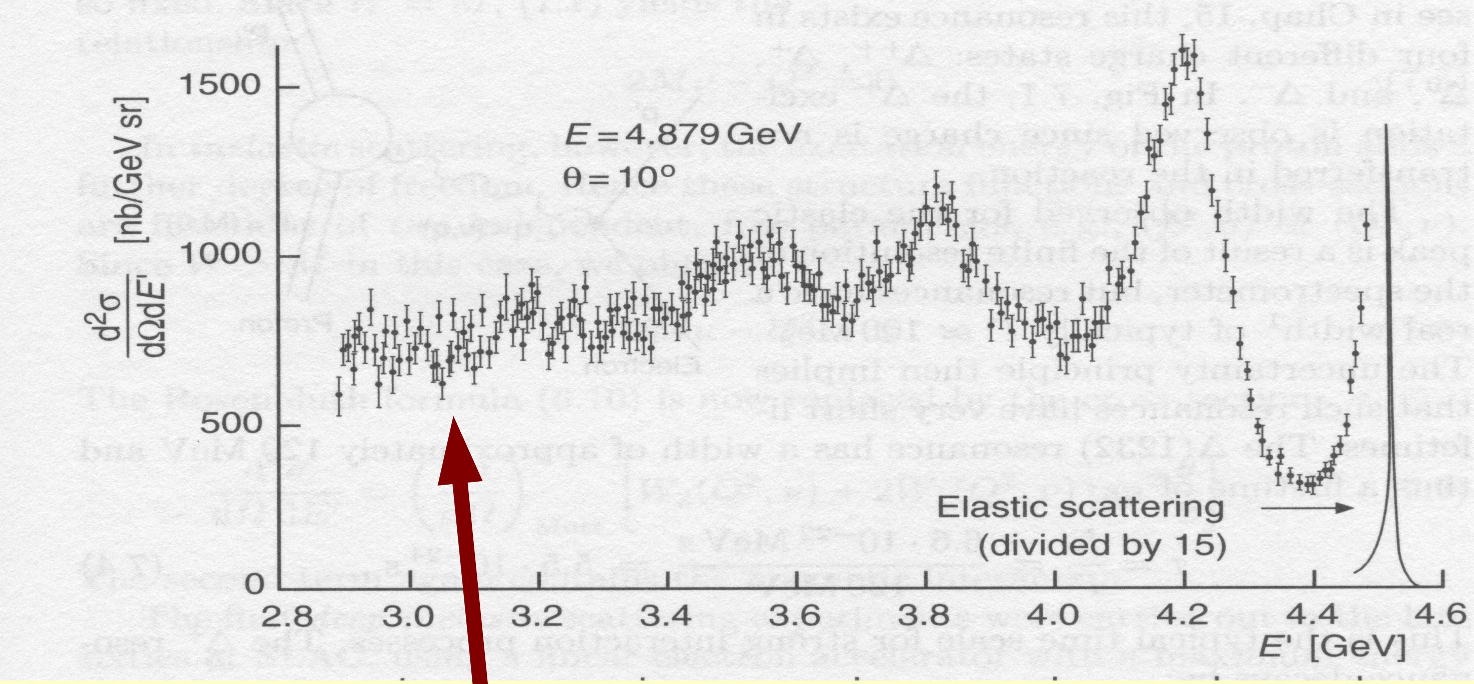
For the elastic scattering events, the diffraction pattern does not have a constant intensity. The proton is <u>NOT</u> a zero-size, point-like particle!



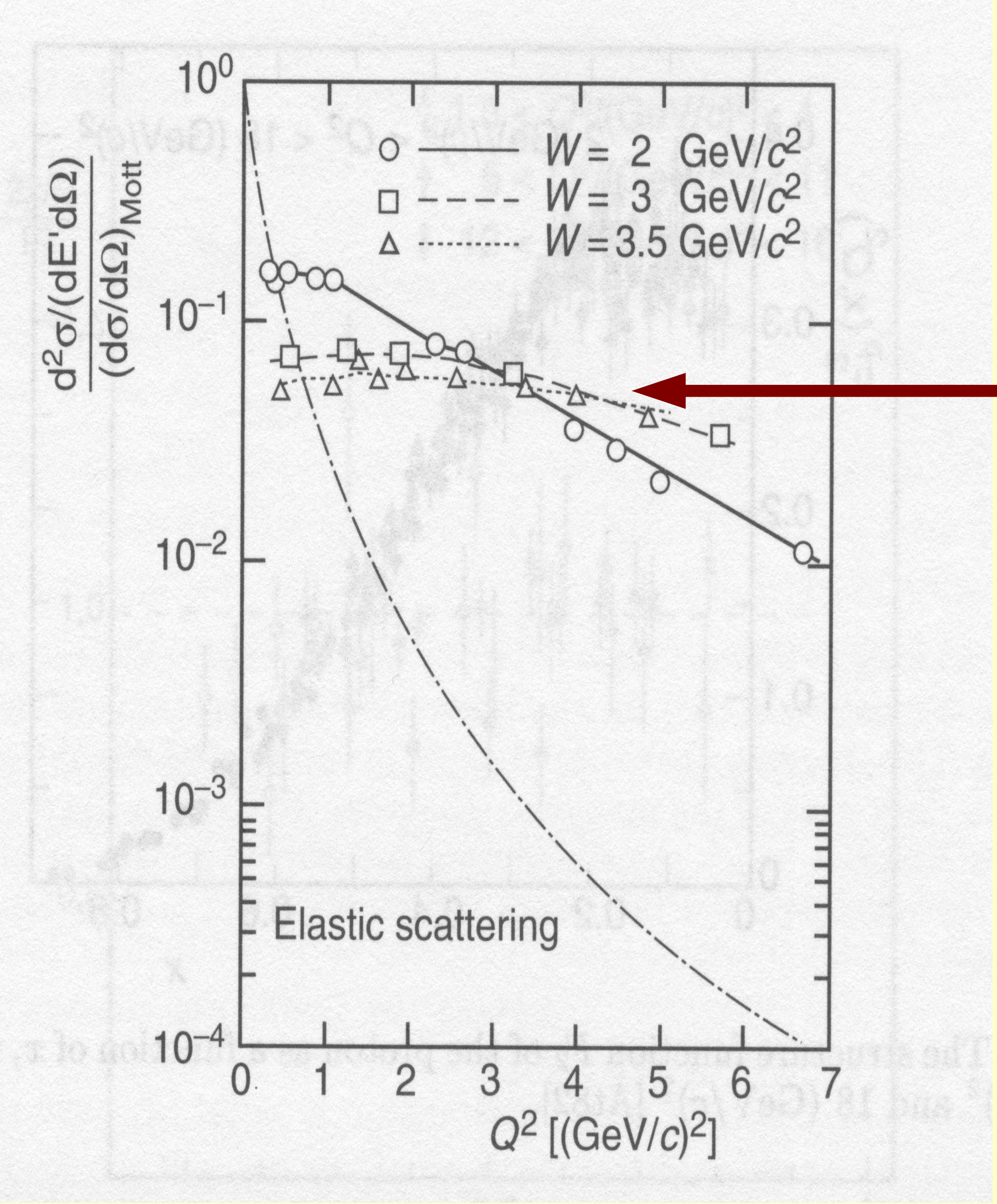


These peaks correspond to inelastic scattering where the target proton is left in an excited state (vibrating, rotating). The electron energy is less because it has given up some energy to excite the proton.

The existence of excited states indicates that the proton must have some internal parts that can be boosted up to higher energy levels.



This is the "deep inelastic scattering" region where the electron has lost a lot of energy when it scatters from the proton. When we plot a graph of the diffraction pattern for these scattering events, we get the following:



For the deep inelastic scattering events, we have a nearly constant diffraction pattern.

This indicates that we are scattering from point-like objects inside the proton. These are quarks.

(called "scaling" in particle physics)

This work won the 1990 Nobel Prize in physics for Richard Taylor, Henry Kendall and Jerome Friedman